The Effect of Measurement Preprocessing in the Gravity-Aided Navigation

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Abstract

The paper analyzes the effect of preliminary processing of gravity measurements on the accuracy of the marine gravity-aided navigation. The preliminary processing of the measurements is implemented in the filtering and smoothing modes. Obtained results are illustrated by a one-dimensional example of gravity-aided navigation problem.

Keywords

Accuracy analysis · Filtering and smoothing · Gravity-aided navigation · Preliminary measurement processing

1 Introduction

Map-aided navigation systems using geophysical fields for a position update are employed for a wide class of vehicles (Stepanov 1998; Bergman 1999). The vehicle position is updating through the comparison of the measured and reference samples (profiles) of the field along the vehicle trajectory. In marine navigation it is common to use the field of Earth gravity anomalies (GA), since it is stable in time and can be measured using well-developed inertial sensors: high-precision accelerometers and gravimeters (Bishop 2002; Sokolov et al. 2019).

However, the measurement of the gravitational field onboard the moving vehicles has its own peculiarities: the signal measured by the gravimeter consists of both GA and inertial accelerations of the object, considered as errors. There are two possible ways for solve the gravity-aiding problem: use raw measurements and consider the detailed error model in the map-aiding algorithm or use the preprocessed measured samples in which the GA along the vehicle trajectory is estimated some way. In the second case, often used in practice, the measurements of the gravimeter are pre-processed (Wu et al. 2017). After that, the amplitude of the measurement errors of the GA significantly decreases, which allows us to simplify the map-aiding algorithm. At the same time, as it is shown in (Nosov and Stepanov 2018), the preliminary processing of measurements and the subsequent simplification of the algorithm can lead to an increase in the navigation error.

This paper analyzes the effect of preliminary processing of gravity measurements on the accuracy of the marine gravity-aided navigation. It continues the authors' study on a similar problem for underwater terrain-aided navigation in a presence of a white noise measurement error (Nosov and Stepanov 2018).

2 Optimal and Suboptimal Solutions of the Gravity-Aided Navigation Problem

Following (Nosov and Stepanov 2018), we consider a gravity-aided navigation problem in the simplest formulation, namely, we need to estimate a constant scalar random variable Δ (for example, a navigation system (NS) error from one of the coordinates) using a previously constructed GA map. This problem can be stated in Bayesian framework as follows (Stepanov 1998; Bergman 1999): to estimate the unknown parameter Eq. (1) using p scalar

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measurements Eq. (2)

$$\Delta_i = \Delta_{i-1} = \Delta, \tag{1}$$

$$y_i = \phi\left(x_i - \Delta\right) + \varepsilon_i = \phi_i\left(\Delta\right) + \varepsilon_i, \tag{2}$$

where x_i are coordinates, provided by the reference navigation system, e.g. inertial one, in a discrete set of points; ε_i are random measurement errors; $i = \overline{1 \ldots p}$; $\phi(\bullet)$ is a exactly known function (map) describing the dependence of the field on coordinate. For simplicity, we consider Δ and ε_i as Gaussian with known stochastic properties.

To solve this problem in optimal way, the well-known numerical point-mass and sequential Monte-Carlo methods are used (Stepanov 1998; Bergman 1999; Nordlund 2002; Gustafsson et al. 2002). However complex behaviour of measurement errors ε_i which in turn requires high-dimension model makes the estimation algorithm implementation computationally expensive, because nonlinear Bayesian estimation methods are subject to "curse of dimensionality" (Daum and Huang 2003). In this case it is suitable to pre-process measurements to estimate the GA along the vehicle trajectory, i.e. the value of $\phi_i(\Delta)$. The measurements Eq. (2) are represented as a sum of some random process which describes the field values $g_i \equiv \phi(x_i - \Delta)$ and the error same as above $y_i = g_i + \varepsilon_i$. The problem of estimating g_i is linear one, which allows the use of Kalman-type filtering and smoothing algorithms (Kalman 1960; Gelb et al. 1974; Peshekhonov and Stepanov 2017).

After the preliminary processing, the measurements used to solve the navigation problem can be written as follows:

$$\hat{y}_k = \phi\left(x_k - \Delta\right) + \varsigma_k, \tag{3}$$

where $\hat{y}_k$ is the estimate, provided by the preliminary filtering or smoothing algorithm; ς_k is the corresponding estimation error; $k = \overline{1 \ldots n}$.

Note that preliminary processing in itself does not make the algorithm for estimating Δ simpler. Moreover, if we use the whole set of measurements Y_p after the pre-processing and take a proper account of the estimation errors, we can show that the estimation accuracy of Δ will remain at the same level. However, since preliminary processing significantly reduces the measurement errors it becomes possible to reducethe number of measurements used to estimate Δ from p to n. This is achieved by increasing the interval Δt_1 for measurements Eq. (2) compared with the interval Δt_2 for measurements Eq. (3). Furthermore if the interval between measurements Eq. (3) is chosen to exceed the correlation interval for the error ς_k, then its model can be approximated by discrete white noise, the level of which will correspond to the solution of a filtering problem or a smoothing one. This simplifies the model used in nonlinear algorithm, since there is no need to describe the complex behavior of measurement errors.

As indicated in the introduction, such a two-stage scheme for gravity-aided navigation algorithm can lead to an increase in positioning errors. To evaluate losses in accuracy, we will use the procedure based on comparing the unconditional calculated and real root-mean square errors (RMSE) for optimal and suboptimal (two-stage) algorithms (Nosov and Stepanov 2018).

3 Accuracy Analysis Example

Let us consider example of the gravity-aided navigation problem and compare the unconditional RMS errors for optimal and suboptimal (two-stage) algorithms. We have to specify stochastic models for random process which describes the GA values and measurement errors. For the GA profile $\tilde{g}(t)$ along a rectilinear trajectory we use the Jordan model in the form of a stationary process (Jordan 1972). Its correlation function can be written as follows:

$$K_{\tilde{g}}\left(\rho\right) = \sigma_{\tilde{g}}^2 \left(1 + \alpha\rho - \frac{\left(\alpha\rho\right)^2}{2}\right) e^{-\alpha\rho}, \tag{4}$$

where $\rho \geq 0$. The corresponding shaping filter is written as (Koshaev and Stepanov 2010; Peshekhonov and Stepanov 2017):

$$\begin{cases} \dot{x}_1 = -\beta x_1 + x_2, \\ \dot{x}_2 = -\beta x_2 + x_3, \\ \dot{x}_3 = -\beta x_3 + q_{\tilde{g}} w_{\tilde{g}}, \end{cases} \\ \tilde{g} = -\beta\zeta x_1 + x_2. \tag{5}$$

In Eqs. (4) and (5) ρ is a distance along the trajectory; α– inverse value of the correlation window; $\beta = V\sigma_{\partial\tilde{g}/\partial\rho}/\sqrt{2}\sigma_{\tilde{g}}$; V is the vessel speed; $\sigma_{\partial\tilde{g}/\partial\rho}$ is the parameter defining GA spatial variability; $q_{\tilde{g}} w_{\tilde{g}}$ is forcing white noise with power-spectrum density (PSD) $q_{g}^2 = 10\beta^3\sigma_{\tilde{g}}^2$; $\sigma_{\tilde{g}}^2$ is the GA variance; and $\zeta = \left(\sqrt{5} - 1\right)/\sqrt{5}$ is the dimensionless coefficient.

To describe the errors of GA measurements on a sea vessel, we consider vertical movement due to heaving and white-noise error. The model describing vertical accelerations a_v can be represented in the form:

$$\begin{cases} \dot{x}_4 = x_5, \\ \dot{x}_5 = x_6, \\ \dot{x}_6 = -b_3 x_4 - b_2 x_5 - b_1 x_6 + w_v, \end{cases} \\ a_v = x_6, \tag{6}$$

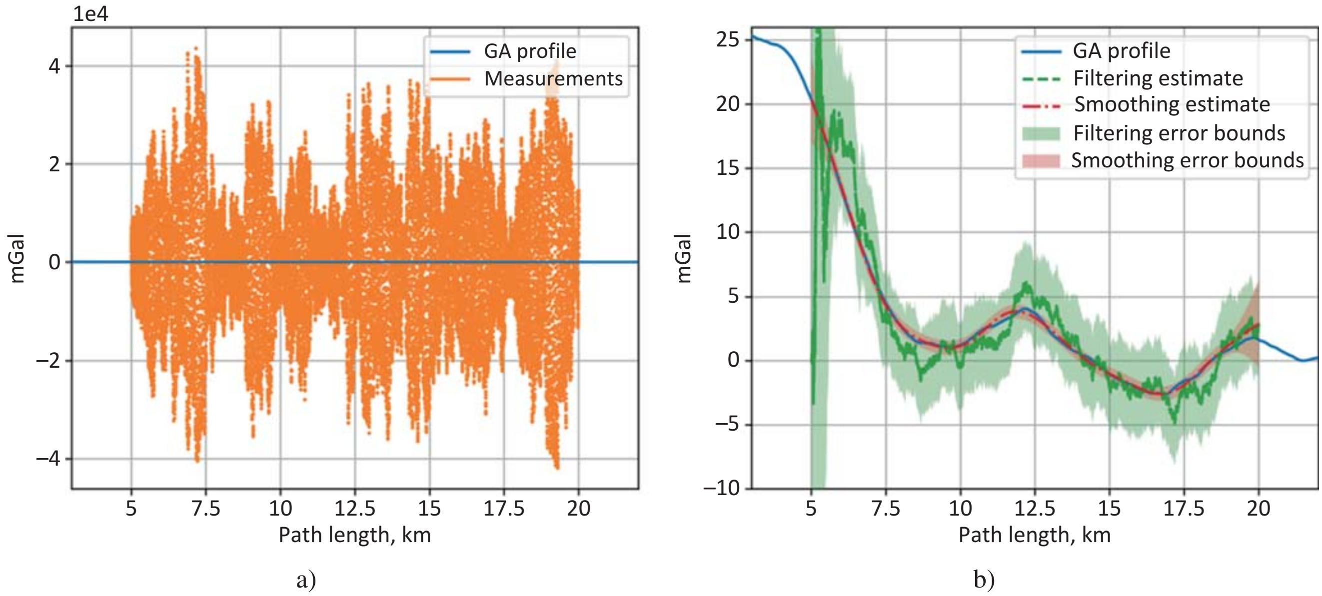

Fig. 1 Example of the GA measurements and results of preliminary processing

where $b_3 = (\lambda^2 + \mu^2)\gamma$; $b_2 = \lambda^2 + \mu^2 + 2\mu\gamma$; $b_1 = 2\mu + \gamma$; $C = \sigma_v \sqrt{2b_3\,(b_1 b_2 - b_3)\,/b_1}$, w_v is white noise of unit PSD; σ_v is the RMS value of vertical movement x_4; λ is the prevailing heaving frequency; μ is the coefficient of heaving irregularity; and γ is the dimensionless coefficient.

In this case, the gravimeter measurements can be written as

$$y = \tilde{g} + a_v + v_{gr} = -\beta\zeta x_1 + x_2 + x_6 + v_{gr}, \quad (7)$$

where v_{gr} is the white-noise measurement error with a known PSD.

Typical parameters for marine gravimetric survey were chosen for modeling the problem according to the discrete representation of Eqs. (5)–(7): the RMS value of the GA derivative was set to be 3 mGal/km, the period of vertical displacements was 7 s, and their RMS value was 0.2 m. Gravimeter measurements in the form Eq. (7) were simulated on a fixed section of several 30 km length GA profiles with coordinates (5,000 − 20,000) m. With spatial interval of 1 m it gives $N = 15{,}000$ raw measurements presented in Fig. 1a. Note that although the RMS value of the sea vessel vertical displacements is only 0.2 m, the RMS error of field measurements exceeds 14,700 mGal. It is obvious that against the background of such errors, it is impossible to estimate of the GA without processing. Figure 1b shows the results of preliminary processing in filtering and smoothing modes, as well as 3σ bounds corresponding to the estimates. Values were obtained using the Kalman filter and the Rauch–Tung–Striebel smoother based on models Eqs. (5)–(7).

Using the GA estimates presented in Fig. 1b we can simplify the map-aiding by replacing model Eq. (6) in nonlinear part of the algorithm with white noise error model. It is feasible by subsampling the estimates, corrupted by correlated pre-processing errors.

To select a subsampling interval, we use the correlation functions of pre-processing errors. The choice of subsampling interval based on value of 0.3 for the normalized correlation function. Under this condition, the spatial intervals for the pre-processed measurements were approximately 1,200 and 800 m for the filtering and smoothing modes, respectively. The variance of the residual discrete white noise error was selected corresponding to the variance of the GA estimation error calculated during the pre-processing.

The raw and pre-processed measurements, represented by Eqs. (2) and (3) were used in optimal and suboptimal algorithms, respectively, to simulate the solution of the gravity-aided navigation problem. In the optimal algorithm, a four-dimensional state vector including the Δ and vector Eq. (6) was estimated. The estimate was calculated as the mathematical expectation of the conditional probability density function (p.d.f). In the same time suboptimal algorithms estimated only the Δ using pre-processed measurements. For calculations in both cases, we used the point-mass method with the number of nodes $L = 3000$, which was supplemented with the Rao-Blackwellization procedure for the optimal algorithm (Stepanov and Toropov 2015). The a priori RMS position error was set at 700 m.

Figure 2 shows the examples of p.d.f. graphs for each algorithm. Orange lines indicate the true error of the NS,

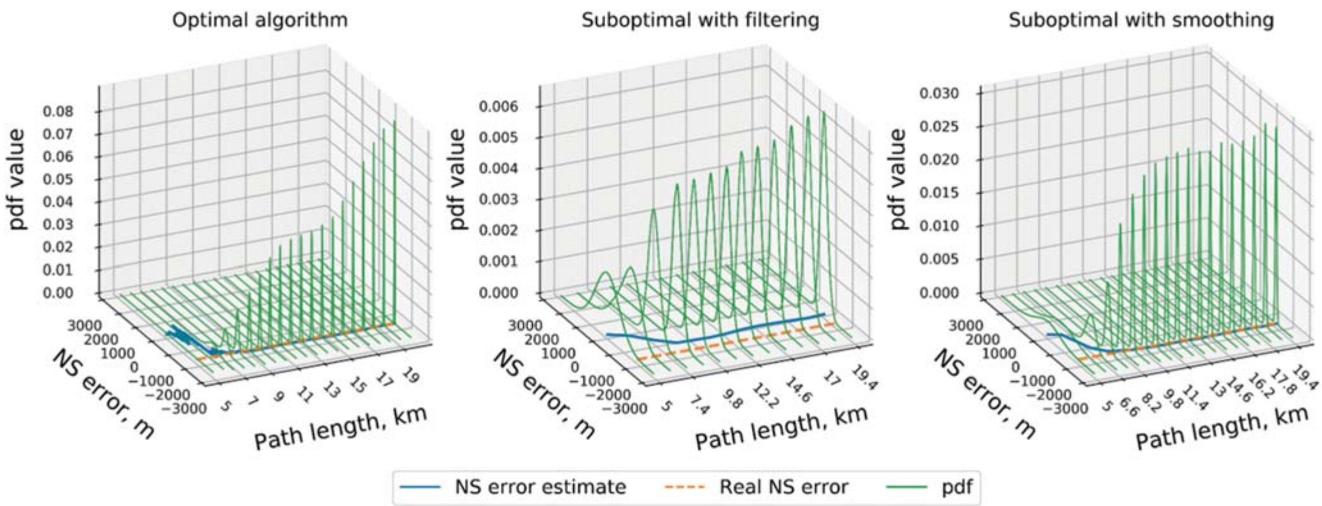

Fig. 2 Examples of p.d.f. for optimal and suboptimal algorithms

Table 1 Real RMSEs and calculation time for optimal and two-stage algorithms

Algorithm	Number of measurements in nonlinear algorithm	RMSE, m	Calc. time, s
Optimal	15,000	≈55	30.2
Suboptimal with filtering	12 (1,200 m subsampling)	≈530	2.1
Suboptimal with smoothing	18 (800 m subsampling)	≈70	3.2

blue ones indicate its estimates obtained in the gravity-aiding algorithm, green ones indicate graphs of a posteriori p.d.f. depending on the distance covered.

Graphs below show calculated and real unconditional RMSEs for the algorithms under study. They were calculated using 250 Monte Carlo simulations. Table 1 contains results including the values of real RMSE obtained at the end of the observation for n measurements and mean calculation time on the reference computer.

From the Fig. 3 and table above, it is obvious that the two-stage suboptimal algorithm with preliminary smoothing is comparable to the optimal algorithm in accuracy: when it was applied, the unconditional real RMSE of these algorithms were in 50–70 m range. In addition, calculated RMSE provided by the suboptimal algorithm with smoothing is close to the real values.

Algorithm with preliminary filtering of measurements underperforms both in accuracy and identity of real and calculated accuracy characteristics. Besides the smaller number of measurements used and greater variance of the estimation error, this can be explained by the presence of a phase delay in the field estimates produced by the filter. Although this algorithm is the fastest, low accuracy precludes its use.

As for the amount of calculations, based on time averaging of 250 runs of algorithms on the test computer, we determined that it took 30 s to process all measurements in the optimal algorithm and an order of magnitude less, that is, 3 s, for the two-stage algorithm which includes preliminary smoothing.

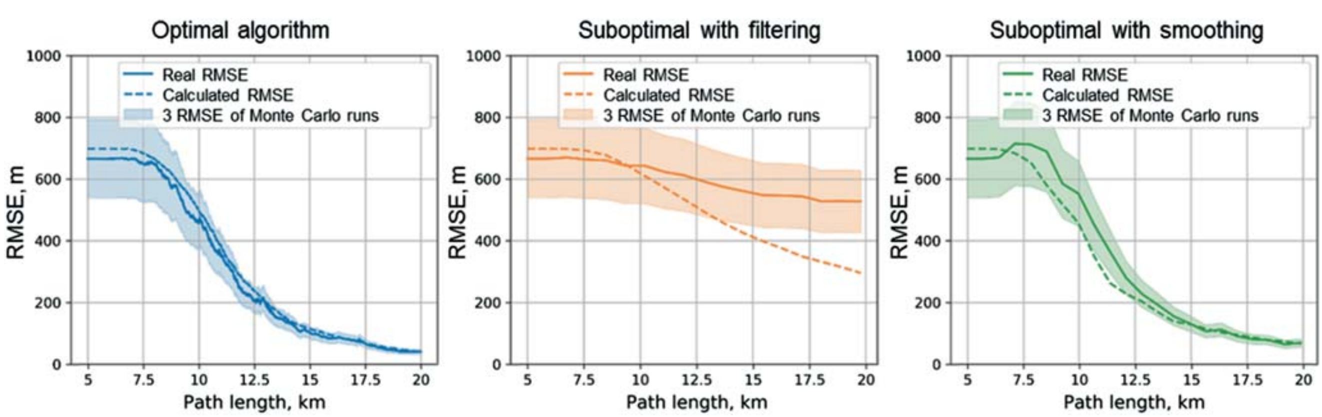

Fig. 3 Real and calculated unconditional RMSEs of the optimal and suboptimal algorithms

4 Conclusion

The effect of preliminary gravity measurement processing on the accuracy of gravity-aided navigation has been analysed. It is based on comparison unconditional real RMS estimation errors for the two-stage suboptimal algorithms that use the measurement pre-processing with potential accuracy achieved by optimal Bayesian algorithm. The preliminary processing of the measurements is implemented in the filtering and smoothing modes.

The example of gravity-aided navigation problem has been considered. It has been shown that the accuracy of the suboptimal algorithm with preliminary smoothing is close to the potential one, and the amount of calculations has been reduced by an order of magnitude in comparison with optimal algorithm.

In the further research, it is supposed to consider map errors model and generalize the results to the case of update two-dimensional position of the vehicle.

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