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Nomenclature

Latin

A	surface of a volume
A	frontal area of bubble
$A^{\alpha\beta}$	surface metric tensor (Aris, 1962)
$\mathbb{A}$	turbulence anisotropy tensor
A_d	projected area of a typical particle
A_i	mathematical surface between A_1 and A_2
A_i	surface area
A_k	surface bounding the interfacial region and adjacent to phase k
A_m	surface of fixed mass volume
A_p	projected area of a particle
a	cross sectional radius of cap or slug bubble
a_c^i	mobility of the fluid at the interface
a_i	interfacial area concentration
a_{sk} , a_{tk}	isentropic and isothermal sound velocities based on the average thermodynamic properties
B_d	volume of a typical particle
B_S	balance at an interface
B_V	balance in each phase
b_k^{Γ} , b_k^M , b_k^E	Transport coefficients associated with interfacial

	transfer of mass, momentum and energy
C	wave velocity
C	constant
C_D	drag coefficient
$C_{D\infty}$	ideal drag coefficient
C_g	variable defined by $\sqrt{2g\Delta\rho/\rho_f}$
C_{hk}	distribution parameter
C_{hm}	mixture-enthalpy-distribution parameter
C_i	closed curve on an interface
C_K	kinematic wave velocity
C_{LW}	coefficient of lift force caused by slanted wake
C_M	virtual mass constant
C_T	adjustable parameter
C_{vk}	distribution parameter
C_{vm}	virtual volume coefficient
C_{vm}	mixture-momentum-distribution parameter
C_τ	distribution parameter
$C_{\psi k}$	distribution parameter for flux
C^i	shape factor
C_0	distribution parameter
C_∞	propagation velocity
C_∞	asymptotic value of distribution parameter
c_k	mass concentration of phase k
c_{pk}, c_{vk}	specific heat at constant pressure and density based on averaged properties
D	hydraulic-equivalent diameter
D^*	length scale ratio
D_b	bubble diameter
D_{bc}	critical bubble size
D_{c1}^*	ratio of D_{crit} to D_{Sm1}
$D_{c,max}$	maximum diameter of stable bubble
D_{crit}	volume-equivalent diameter of a bubble at boundary between groups 1 and 2
$D_{d,max}$	maximum distorted bubble limit

D_d^*	ratio of bubble diameter to bubble diameter at distorted bubble limit
D_e	volume-equivalent diameter of a fluid particle
D_e	eddy diameter
D_E	effective diameter of mixture volume that contains one bubble
D_H	hydraulic-equivalent diameter
D_H^*	non-dimensional hydraulic-equivalent diameter
D_k	diffusion coefficient
$\mathbb{D}_k$	total deformation tensor of phase k
$\mathbb{D}_{kb}$	bulk deformation tensor
$\mathbb{D}_{ki}$	interfacial extra deformation tensor
D_k^α	drift coefficient
D_{Sm}	Sauter mean diameter
D_s	surface-equivalent diameter of a fluid particle
d_B	bubble diameter
$\widehat{d_B}$	cross-sectional mean diameter of bubbles
E_B	average energy required for bubble breakup
E_d	area fraction of liquid entrained in gas core from total liquid area at any cross section
E_e	average energy of a single eddy
E_k	total energy gain through interfaces for phase k
E_m	mixture total energy source from interfaces
E_m^H	mixture energy gain due to changes in mean curvature
Eo	Eötvös number
$\widehat{e_k}$, $\widehat{e_{ki}}$	weighted mean virtual internal energy (with turbulent kinetic energy included) at the bulk phase and at the interfaces
$F(\boldsymbol{x},t)$	general function
$\boldsymbol{F}^B$	Basset force
$\boldsymbol{F}^D$	standard drag force
$\boldsymbol{F}^L$	lift force
$\boldsymbol{F}^T$	turbulent dispersion force
$\boldsymbol{F}^V$	virtual mass force

$\boldsymbol{F}^W$	wall lift force
F_D	drag force
F_k, $\mathbb{F}_k$	general function associated with phase k
$f(\boldsymbol{x},t)$	function for interface position
$f(\boldsymbol{x},t,\boldsymbol{\xi})$	molecular density function
f	collision frequency
f	friction factor
f^*	correction factor for drag coefficient
f_i	interfacial friction factor
f_k	Helmholtz potential
$f_{kn}(\boldsymbol{x},t,\boldsymbol{\xi})$	particle density function of the n^{th}-kind particles
f_{TW}	two-phase friction factor
G	mass velocity
G	cap bubble thickness
G_s	non-dimensional velocity gradient
$\boldsymbol{g}$	gravity field
$\boldsymbol{g}_k$	body force field
g_k, $\widehat{g_k}$, $\widehat{g_{ki}}$	Gibbs free energy: local instant, bulk mean and interfacial mean values
g_{ln}	space metric tensor (Aris, 1962)
g_N	normal gravitational acceleration
H_{21}, $\overline{\overline{H_{21}}}$	local instant and averaged mean curvature ($\overline{\overline{H_{21}}} > 0$ if phase 2 is the dispersed phase)
h	bubble height
h_1, h_2	average thickness of upper (1) and lower (2) fluid layers
$\widehat{h_k}$, $\widehat{h_{ki}}$	weighted mean virtual enthalpy (with turbulent kinetic energy included) at the bulk phase and at the interfaces
h_m	mixture virtual enthalpy
$\mathbb{I}$	unit tensor
I_k	interfacial source term in the balance equations for phase *k*
I_m	interfacial source term for mixture balance equations
I_{ka}, I_{ma}	interfacial source terms in the shock conditions for

	phase k and for mixture
i_k, $\widehat{i_k}$	local instant and mean enthalpies
$\widehat{i_{ki}}$	mean enthalpy of phase k at interfaces
i_m	mixture enthalpy
i_a	local instant surface enthalpy per area
$\mathcal{J}$	flux
$\mathcal{J}^D$	drift flux
$\mathcal{J}_a$	line flux for interface
$\mathcal{J}_k$	surface flux for phase k
$\mathcal{J}_k^T$, $\mathcal{J}^T$	turbulent fluxes
J_k, J_m	Jacobians based on macroscopic field
$\boldsymbol{j}_k$, $\boldsymbol{j}$	volumetric fluxes of phase k and mixture
j^*	non-dimensional mixture volumetric flux
j^+	non-dimensional mixture volumetric flux
K	constant
K_k	thermal conductivity
$\mathbb{K}_k$	thermal conductivity tensor
K_k^T	turbulent conductivity
K_k^{T*}	thermal mixing length coefficient
k	wave number
k^{SI}	turbulent kinetic energy due to shear-induce turublence
k_e	wave number of eddy
L	pitch of slug unit
L_b	cylindrical bubble length
$1/L_j$	area concentration of j^{th}-interface
$1/L_s$	total area concentration
L_T	mean traveling distance between two bubbles for one collision
L_W	effective wake length
l	mixing length
l_B	mixing length due to bubble-induced turbulence
l_{SP}	mixing length of single-phase flow
l_{TP}	mixing length of two-phase flow
m_e	mass per a single eddy

$\dot{m}_k, \overline{\overline{\dot{m}_k}}$	local instant and mean mass transfer rates per unit area (mass loss)
M	Morton number
M_F	frictional pressure gradient in multi-particle system
$M_{F\infty}$	frictional pressure gradient in single particle system
$\boldsymbol{M}_{ik}$	generalized interfacial drag
M_k, M_s	state density functions for phase k and interface
$\boldsymbol{M}_k, \boldsymbol{M}_m$	momentum sources for phase k and mixture
$\boldsymbol{M}_m^H$	force due to changes in mean curvature
$\boldsymbol{M}_k^n, \boldsymbol{M}_k^t, \boldsymbol{M}_k^d$	form, skin and total drag forces
$\boldsymbol{M}_{\tau m}$	force associated with mixture transverse stress gradient
$\boldsymbol{N}$	unit normal vector to a curve on an interface
N	number of samples
N_b	number of bubbles
N_D	drift number
N_{drag}	drag number
N_e	number of eddies of wave number k_e per volume of fluid
N_{Ec}	Eckert number
N_{Eu}	Euler number
N_{Fr}	Froude number
N_i	converted enthalpy ratio
N_{Jk}	Jakob number
N_{pch}	phase change number
N_{pch}^i	interfacial phase change effect number
N_{Pe}	Peclet number
$N_{Pr\,k}^T$	turbulent Prandtl number
N_q	interface heating number
N_{Re}	Reynolds number
N_{Re}^i	interfacial Reynolds number
N_{Sl}	Strouhal number
N_{We}	Weber number
N_σ	surface tension number

N_{Pr}	Prandtl number
N_W	number of bubbles inside effective volume
N_μ	viscosity number
N_ρ	density ratio
n	fluid particle number per unit mixture volume
$\boldsymbol{n}$	unit normal vector
n_b	bubble number density
n_e	number of eddies of wave number per volume of two-phase mixture
$\boldsymbol{n}_k$	outward unit normal vector for phase k
P^{SI}	production of shear-induced turbulence
P_C	probability for a bubble to move toward neighboring bubble
$\overline{\mathcal{P}_k}$	partial pressure tensor
P_i	interfacial wetted perimeter
P_{wf}	wall wetted perimeter
p	pressure
p_c	critical pressure
p_k, $\overline{\overline{p_k}}$, $\overline{\overline{p_{ki}}}$	partial, bulk mean and interfacial mean pressure
p_m	mixture pressure
$\boldsymbol{q}$	heat flux
$\boldsymbol{q}^D$	diffusion (drift) heat flux
$\overline{\overline{\boldsymbol{q}_k}}$, $\boldsymbol{q}_k^T$	mean conduction and turbulent heat fluxes
$\overline{\boldsymbol{q}}$, $\boldsymbol{q}_k^T$	mixture conduction and turbulent heat fluxes
$\dot{q}_k$	local instant body heating
$\overline{\overline{q_k''}}$	average heat transfer pert interfacial area (energy gain)
$\boldsymbol{q}_k^C$	mean conduction heat flux
R	ideal gas constant
R	radius of a pipe
R	radius of curvature
R^+	variable defined by Rv_f^*/ν_f
$\overline{\overline{R_d}}$	mean radius of fluid particles
R_j	particle number source and sink rate

R_w	tube radius
Re	Reynolds number
$(Re)_d$	particle Reynolds number
r	radial coordinate
r_d^*	non-dimensional radius
S_B, S_C	surface available to collision
S_j	particle source and sink rates per unit mixture volume due to j-th particle interactions such as disintegration or coalescence
S_{ph}	particle source and sink rates per unit mixture volume due to phase change
s	entropy
s_a	surface entropy per area
$\widehat{s_k}$, $\widehat{s_{ki}}$	weighted mean entropy at bulk phase and at interfaces
s_m	mixture entropy
T	temperature
$T_i, \overline{\overline{T_i}}$	instant and mean interface temperature
$\overline{\overline{T_k}}$, $\overline{\overline{T_{ki}}}$	mean temperature at bulk phase and at interface
$\mathcal{T}_k$	stress tensor
t	time
t_C	time required for bubble coalescence
t_j	time when the j^{th}-interface passes the point
t_α^m (or t_α)	hybrid tensor of interface, see Aris (1962)
$\boldsymbol{U}$	velocity of shock in mixture
U_0	velocity of stream
U_B, U_C	volume available to collision
u	internal energy
u_a	surface energy per area
u_b	mean fluctuation velocity
u_B, u_C	bubble velocity
u_e	eddy velocity
$\widehat{u_k}$, $\widehat{u_{ki}}$	weighted mean internal energy at bulk phase and at interfaces
u_m	mixture internal energy

u_{rW} averaged relative velocity between leading bubble and bubble in wake region
u_t root-mean-square approaching velocity of two bubbles
$u_{t,crit}$ critical fluctuation velocity
V volume
$\dot{V}$ time derivative of volume V
V_c critical bubble volume
V_{gj}^{+} non-dimensional drift velocity
V_i interfacial region
$\boldsymbol{V}_{kj}$ drift velocity
$\boldsymbol{V}_{km}$ diffusion velocity
V_m fixed mass volume
V_s^* ratio of $V_{s,min}$ to $V_{s,max}$
V_W effective wake volume
V_{1p} peak bubble volume in group 1
$\boldsymbol{v}$ velocity
v_f' liquid velocity fluctuation independent of bubble agitation
v_f'' liquid velocity fluctuation dependent on bubble agitation
v_f^* friction velocity
$\boldsymbol{v}_g$ average center-of-volume velocity of dispersed phase
$\boldsymbol{v}_i$ velocity of interface
$\widehat{\boldsymbol{v}_k}$, $\widehat{\boldsymbol{v}_{ki}}$ weighted mean velocity at bulk phase and at interfaces
$\widehat{\left(v_k'\right)^2}/2$ mean turbulent kinetic energy
$\boldsymbol{v}_m$ mixture center of mass velocity
$\boldsymbol{v}_{pm}$ average local particle velocity weighted by particle number
$\boldsymbol{v}_r$ relative velocity
$\overline{v_r}$ difference between area averaged mean velocities of phases
$v_{r\infty}$ relative velocity of a single particle in an infinite medium

$\boldsymbol{v}_s$	velocity of interfacial particles
W_{ki}^T	work due to fluctuations in drag forces
We	Weber number
We_{crit}	critical Weber number
$\boldsymbol{X}$	convective coordinates
$\boldsymbol{x}$	spatial coordinates
x	spatial coordinate
y	spatial coordinate
y^+	variable defined by yv_f^* / ν_f
z	spatial coordinate

Greek

α_b	void fraction in slug bubble section
α_{core}	ratio of liquid-film cross-sectional area to total cross-sectional area
α_d	average overall void fraction
α_{drop}	ratio of cross-sectional area of drops to cross-sectional area of core
$\alpha_{g,crit}$	critical void fraction when center bubble cannot pass through free space among neighboring bubbles
$\alpha_{g,max}$	maximum void fraction
α_k	time (void) fraction of phase *k*
β	ratio of mixing length and width of wake
β_C	variable to take account of overlap of excluded volume
β_k	thermal expansivity based on averaged properties
Γ	constant
Γ_k	mass generation for phase *k*
γ	constant
γ_k	ratio of specific heats
Δ_a	interfacial entropy generation per area
Δ_k	entropy generation for phase *k*
$\Delta \dot{m}_{12}$	inter-group mass transfer rates from group 1 to group 2
Δt	time interval of averaging
Δt_B	time interval to drive daughter bubble apart with

	characteristic length of D_b
Δt_C	time interval for one collision
Δt_k, Δt_s	time intervals associated with phase *k* and interfaces
Δt_W	average time interval for a bubble in wake region to catch up with preceding bubble
δ	thickness of interface
δ	film thickness
δ'	collective parameter
δ_{crit}	critical film thickness where rapture occurs
δ_{init}	initial film thickness
δp_k	pressure deviation from saturation pressure
$\delta\mu$	volume element in μ space
ε	energy dissipation rate per unit mass
2ε (or $2\varepsilon_j$)	time associated with the j^{th}-interface
ε^{SI}	dissipation of shear-induced turbulence
ε', ε''	eddy diffusivity
η_{ph}	rate of volume generated by nucleation source per unit mixture volume
η_0	amplitude
Θ	contact angle
θ	angle in cylindrical coordinates
θ_w	wake angle
κ_{fr}	variable defined by $1 - exp\left(-C_{fr}V_s^{*1/2}/D^{1/2}\right)$
κ_{Sk}, κ_{Tk}	isentropic and isothermal compressibilities of phase *k*
Λ_k	interfacial thermal energy transfer term in the averaged equation
λ	wavelength
λ	constant
λ_B	breakup efficiency
λ_C	coalescence efficiency
λ_c	critical wavelength
λ_k	bulk viscosity
μ	viscosity
$\overline{\overline{\mu_k}}$, μ_k^T	mean molecular and turbulent viscosities

μ_k^{T*}	mixing length coefficient
μ_m	mixture viscosity
ν	kinematic viscosity
ν_t	turbulent kinematic viscosity
$\boldsymbol{\xi}$	particle (phase) velocity in Boltzmann statistical average
ξ	ratio of V_{1p} to V_c
ξ	variable defined by $2\left(1-0.2894D_{c1}^{*3}\right)^2$
ξ	variable defined by P_i/P_{wf}
ρ	density
ρ_a	surface mass per area
$\overline{\rho_k}$, $\overline{\overline{\rho_k}}$	partial and mean densities
ρ_k'	modified density defined by $\rho_k \coth\left(kh_k\right)$
ρ_m	mixture density
σ	surface tension
$\mathcal{T}$	viscous stress tensor
$\mathcal{T}^D$	diffusion (or drift) stress tensor
$\mathcal{T}_f^{BI}$	bubble-induced turbulent stress tensor
$\mathcal{T}_f^{SI}$	shear-induced turbulent stress tensor
$\overline{\mathcal{T}}$, $\mathcal{T}^T$	mixture viscous and turbulent stress tensors
$\overline{\overline{\mathcal{T}_k}}$, $\mathcal{T}_k^T$	average viscous and turbulent stress tensor
$\mathcal{T}_k^\mu$	average viscous stress
$\overline{\overline{\mathcal{T}_{ki}}}$, $\mathcal{T}_{ki}$	interfacial shear stress
τ_C	contact time for two bubbles
τ_i	interfacial shear stress
τ_o	reference time constant
$\boldsymbol{\tau}_{tk}$, $\boldsymbol{\tau}_{nk}$	tangential and normal stresses at interface
τ_{wf}	wall shear
Φ	velocity potential
Φ_k^T	turbulent work effect in enthalpy energy equation
Φ_m^i	interfacial mechanical energy exchange effect in the mixture thermal energy equation
Φ_k^μ	viscous dissipation

Φ_m^μ	mixture viscous dissipation
Φ_m^σ	surface tension effect in the mixture thermal energy equation
ϕ	source term
ϕ_a	interfacial source per area
ϕ_j	source and sink rate for interfacial area concentration
ϕ_k	velocity potential
χ	coefficient accounting for contribution from inter-group transfer
ψ	property of extensive characteristics
ψ	shape factor
$\widehat{\psi}, \widehat{\psi_k}$	mass weighted mean values for mixture and phase k
ψ_a	property per interfacial area
Ω	potential function

Subscripts and Superscripts

a	surface (property per area)
c	continuous phase
d	dispersed phase
f	liquid phase
g	vapor phase
i	interface
j	j^{th}-interface
k	each phase : (k=1 & 2), (k=c & d), (k=f & g)
ki	k^{th}-phase at interfaces
m	mixture (in macroscopic formulation) fixed mass (in local instant formulation)
n	normal to interface
o	reference
RC	random collision
r, θ, z	cylindrical coordinate
sat	saturation
s	surface (surface property per mass) solid phase

SI	surface instability
SO	shearing off
TI	turbulent impact
WE	wake entrainment
t	tangential to interface
w	wall
x, y, z	rectangular coordinate
$+, -$	+ and – side of shock in macroscopic field
$1, 2$	phase 1 and phase 2

Symbols and Operators

$\mathbb{A}$	tensor
$\boldsymbol{A}$	vector
A	scalar
$\boldsymbol{A} \cdot \boldsymbol{B}$	dot product
$\boldsymbol{AB}$	dyadic product of two vectors (=tensor)
$\mathbb{A}:\mathbb{B}$	double dot product of two tensors (=scalar)
$\nabla \cdot$	divergence operator
∇	gradient operator
$\nabla_s \cdot$	surface divergence operator (Aris, 1962)
$(\mathbb{A})^+$	transposed tensor
$\frac{D_k}{Dt}$	$= \frac{\partial}{\partial t} + \widehat{\boldsymbol{v}_k} \cdot \nabla$
$\frac{D}{Dt}$	$= \frac{\partial}{\partial t} + \boldsymbol{v}_m \cdot \nabla$
$\frac{D_c}{Dt}$	$= \frac{\partial}{\partial t} + \boldsymbol{C}_k \cdot \nabla$
$\frac{D_i}{Dt}$	$= \frac{\partial}{\partial t} + \widehat{\boldsymbol{v}_i} \cdot \nabla$
$\frac{d_s}{dt}$	surface convective derivative with $\widehat{\boldsymbol{v}_s}$ (Aris, 1962)
$\overline{F}$	time average
$\overline{F}^{w}$	weighted mean value
$\overline{F}^{w_k}$	k^{th}-phase weighted mean value
$\overline{\overline{F}}$	phase average

$\widehat{\psi_k}$	k^{th}-phase mass weighted mean value
$\widehat{\psi}$	mixture mass weighted mean value
F_k'	fluctuating component with respect to mean value
F_{ki}'	fluctuation component with respect to surface mean value
$\overline{\overline{F_{(i)}}}, \overline{\overline{F_{ki}}}$	surface average
$\widehat{F_{ki}}$	mass flux weighted mean value at interfaces
$(\ )_{,\beta}$	surface covariant derivative (Aris, 1962)
$[\Delta t]_k$	with (k=T,S,1,2) ; sets of time intervals
$\sum_k$	summation on both phases
$\sum_j$	summation on the interfaces passing in Δt at $\boldsymbol{x}$

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