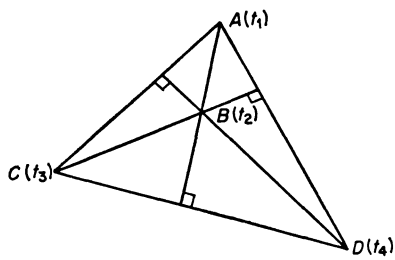
ANSWERS TO SOME EXERCISES

1.3 (i) the integers (ii) positive rationals (iii) rationals.

1.9 (i) no (ii) no

1.12 (ii) $(t_1 + t_2) + (t_3 + t_4) = (t_1 + t_3) + (t_2 + t_4)$

1.14 From **1.13** we see that if AB and CD are perp., so are AC and BD, and also AD and BC, and that any one of the points A, B, C, D, is the orthocentre of the triangle formed by the other three.



1.15 XYZ is a straight line parallel to the axis.

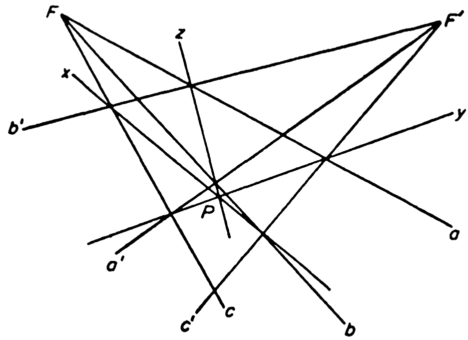
At each point $y = \frac{a}{2}(t_1 + t_2 + t_3 + t_4)$

1.18 (i) $a.(a' + b) = a.b$
(ii) $y.z + z.x + x.y = (y + z).(z + x).(x + y)$
(iii) $(a.b)' = a' + b'$
(iv) $(a + b).(b + c).(c + a') = (a + b).(c + a')$

1.21 If a, b, c are three lines through a point F, and similarly for a' b', c' through F′, and also

$b'c$ and bc' determine the line x
ca' ,, $c'a$,, ,, ,, y
ab' ,, $a'b$,, ,, ,, z

then x, y, z meet in P.



1.22 The dual theorem is also the converse.

1.26 (i) 0 (ii) 1

1.29 (I) no similar form
(II) If A $\geqslant$ B, then B $\leqslant$ A
(III) If A $\geqslant$ B and B $\geqslant$ C, then A $\geqslant$ C

1.30 Both laws apply to both operations.
There is symmetry about a diagonal.

1.31

$\cap$	1	0
1	1	0
0	0	0

2.8 (i) In (6) put A = 1
(ii) .... A = 0
(iii) ...(6D).. A = 1
(iv) .... A = 0

2.10 (i) In (7) put A = 0
(ii) ...(7D).. A = 1

2.13 $0 \cup 0' = 1$ (4)
$0' \cup 0 = 1$ (1)
$0' = 1$ (3)

2.14 Follows from **2.12** and **2.13**.

2.15 (2D), (4), (1D) and (3D).

2.16 (i) $(A \cup B) \cap (A' \cap B')$

$$\begin{aligned}
&= [A \cap (A' \cap B')] \cup [B \cap (A' \cap B')] && (1, 2)\\
&= [(A \cap A') \cap B'] \cup [(B \cap B') \cap A'] && (1, 5D)\\
&= (0 \cap B') \cup (0 \cap A') && (4D)\\
&= 0 \cup 0 && (1D, 6D)\\
&= 0 && (7)
\end{aligned}$$

$(A \cup B) \cup (A' \cap B')$

$$\begin{aligned}
&= A \cup B \cup (B' \cap A') && (5, 1D)\\
&= A \cup (B \cup A') && (11)\\
&= (A \cup A') \cup B && (5)\\
&= 1 \cup B && (4)\\
&= 1 && (1, 6)
\end{aligned}$$

(ii) $A \cap (P \cup Q \cup R)$

$$\begin{aligned}
&= A \cap [(P \cup Q) \cup R] && (5)\\
&= [A \cap (P \cup Q)] \cup (A \cap R) && (2)\\
&= (A \cap P) \cup (A \cap Q) \cup (A \cap R) && (2)
\end{aligned}$$

$(A \cup B) \cap (P \cup Q)$

$$\begin{aligned}
&= [(A \cup B) \cap P] \cup [(A \cup B) \cap Q] && (2)\\
&= (A \cap P) \cup (A \cap Q) \cup (B \cap P) \cup (B \cap Q) && (2, 1, 1D)
\end{aligned}$$

$(A \cup B \cup C)'$

$$\begin{aligned}
&= \{(A \cup B) \cup C\}' && (5)\\
&= (A \cup B)' \cap C' && (12D)\\
&= (A' \cap B') \cap C' && (12D)\\
&= A' \cap B' \cap C' && (5D)
\end{aligned}$$

2.18 (3D) & (4); (2); (1) & (1D); 8.

2.19

$$\begin{aligned}
A \cup B &= A \cup C\\
A' \cap (A \cup B) &= A' \cap (A \cup C) && (IVD)\\
(A' \cap A) \cup (A' \cap B) &= (A' \cap A) \cup (A' \cap C) && (2)\\
0 \cup (A' \cap B) &= 0 \cup (A' \cap C) && (1D, 4D)\\
A' \cap B &= A' \cap C && (3)
\end{aligned}$$

but

$$\begin{aligned}
A \cap B &= A \cap C\\
(A' \cap B) \cup (A \cap B) &= (A' \cap C) \cup (A \cap C) && \textbf{1.27}\ (III), (IV)\\
B \cap (A \cup A') &= C \cap (A \cup A') && (1, 1D, 2)\\
B \cap 1 &= C \cap 1 && (4)\\
B &= C && (3D)
\end{aligned}$$

2.21 (i) (12), (12).
(ii) (12D), (12D), (9).

2.22 (i) $(A' \cap B' \cap C')'$
(ii) $A \cup (B' \cup C')' \cup (B' \cup C \cup D')'$

2.25 (a) $A' \cap B' \cap C$
(b) 1
(c) 0
(d) $(B \cap C) \cup (A' \cap B')$

2.27 (i) (ii) (iii) yes, (iv) no.

2.29 (a) $(X' \cap Y') \cup (Y' \cap Z') \cup (Z' \cap X') \cup (X \cap Y \cap Z)$
(b) $(A \cap B') \cup (A' \cap B)$
(c) $(B \cap C \cap D) \cup (C \cap D \cap A) \cup (D \cap A \cap B)$
$\cup (A \cap B \cap C)$

2.30 There are two ways of choosing the first factor—it is either A_1 or A_1'—and so on.

2.31 (i) $A.B.C + A'.B.C + A'.B'.C + A'.B'.C'$
(ii) $X'.Y'.Z + X'.Y'.Z' + X.Y'.Z' + X'.Y.Z' + X.Y.Z$
(iii) No change is necessary

2.32 (i) A (ii) B' (iii) $A.B$

2.34 (i) $(A + B + C).(A + B' + C)$
(ii) $(X + Y + Z)(X' + Y + Z)(X + Y' + Z)(X + Y + Z')$

2.37
$$\begin{aligned} E_1.E_2 &= (X.Y + X'.Y' + X.Z).(X.Y + X'.Y' + Y'.Z) \\ &= X.Y + X'.Y' + X.Y'.Z \\ &= X'.Y' + X.(Y + Y'.Z) \\ &= X'.Y' + X.(Y + Z) \\ &= X'.Y' + X.Y + X.Z \\ &= E_1 \\ E_1.E_2' &= (X.Y + X'.Y' + X.Z).(X + Y).(X' + Y') \\ &\qquad .(Y + Z') \\ &= (X.Y + X'.Y'.Z').(X.Y' + X'.Y) \\ &= 0 \end{aligned}$$

2.39 (ii) If $A.B' + A'.B = 0$,

then $$A.B' = A'.B = 0$$
$$A + A'.B = A$$
$$A + B = A \quad \text{and similarly} \quad A + B = B$$
so $$A = B$$

2.41 (i) $L = (A + C).(P + R)' + (P + R).(A + C)'$
$M = (B + C).(Q + R) + (Q + R).(B + C)$
$N = C.P'.Q'.R' + R.A'.B'.C'$
(ii) (a) $B'.X + (A'.B + A.B').X' = 0$ if $AB' = 0$
(b) $B.X' + (A + B)X = 0$ if $A = 0$

3.1 $A = C, \quad B = D = E$

3.2 (i) All are equivalent; $n(A) = n(B) = \cdots = n(E) = 3$
(ii) The different uses of '=' in the two statements.

3.4 (i) M' = the soa women
(ii) 1 = ,, ,, human parents

3.5 (a) (i) the soa one's brothers, and oneself, if a male
(ii) ,, ,, rectangles
(b) (i) ,, ,, children
(ii) ,, ,, positive integers

3.7 $A + A'.B = A + B$ (see **2.6**, theorem 11)

3.10 (i)
$$\begin{aligned}
A + B &= B \\
(A + B).B' &= B.B' && \textbf{1.27}\ (\text{IVD}) \\
A.B' + B.B' &= B.B' && (2) \\
A.B' + 0 &= 0 && (4D) \\
A.B' &= 0 \quad \text{and} \quad A \subseteq B && (3), \textbf{3.8}
\end{aligned}$$

(ii)
$$\begin{aligned}
(A.B).A' &= (A.A').B && (5D) \\
&= 0.B && (4D) \\
&= 0 \quad \text{so} \quad (A.B) \subseteq A && (6D), \textbf{3.8}
\end{aligned}$$

(iii)
$$\begin{aligned}
A.(A + B)' &= A.A'.B' && (12D) \\
&= 0 && (4D, 6D)
\end{aligned}$$

(iv)
$$\begin{aligned}
\text{If } A \subseteq B, \quad A.B' &= 0 && \textbf{3.8} \\
B'.(A')' &= 0 && (9, 1D) \\
B' &\subseteq A' && \textbf{3.8}
\end{aligned}$$

(v) $X.Y = X$, dual of **3.9** (i) or
$X.Y' = X.Y.Y' = 0$ (IVD, 4D, 6D)
so $X \subseteq Y$
$$\begin{aligned}
\text{conversely } X &= X.1 && (3D) \\
&= X.(Y + Y') && (4) \\
&= X.Y + X.Y' && (2) \\
&= X.Y + 0 \quad \text{since} \quad X \subseteq Y \\
&= X.Y && (3)
\end{aligned}$$

3.14 (I) $G + G'.S = G + S$

(II) $A.S' + A'.S$ or $(A + S).(A.S)'$

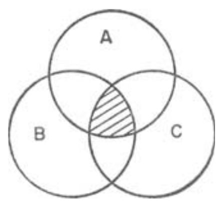
(III) $G'.S + G.S'$

3.15 (i) (ii) and (iv) yes, (iii) no.

3.16 No.

3.17 Uncle Bertrand, Roger, Tom, and Vera.

3.19 (a) (i)



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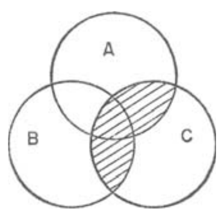

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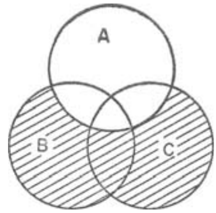
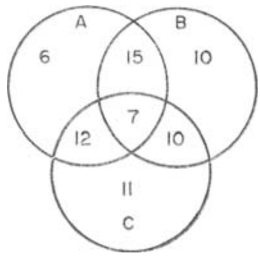


(b) The points P, Q.

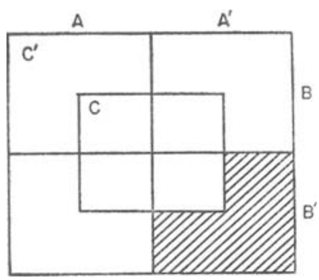
$n(A.B) = 0$ when a and b do not meet
$n(A.B) = 1$,, ,, ,, touch one another

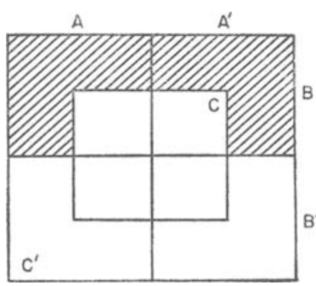
3.20 6 liked apples only



3.22 (i) A′B′C′

(ii) B.C′





3.25 (i) $n(\mathrm{P + Q + R + S}) = n(\mathrm{P}) + n(\mathrm{Q}) + n(\mathrm{R}) + n(\mathrm{S})$
$- n(\mathrm{P.Q}) - n(\mathrm{P.R}) - n(\mathrm{P.S}) - n(\mathrm{Q.R}) - n(\mathrm{Q.S})$
$- n(\mathrm{R.S}) + n(\mathrm{Q.R.S}) + n(\mathrm{R.S.P}) + n(\mathrm{S.P.Q})$
$+ n(\mathrm{P.Q.R})$
$- n(\mathrm{P.Q.R.S})$

3.27 (i) Draw APQ through A

(ii) P on FE to Q on BD by lines parallel to AB
P „ „ „ R „ BC „ „ through A

(iii) $0 < x < 1$; $1 < y < 2$; $1 < z$

correspondence established by

$$y = 1 + x$$
$$z = 1/x$$

3.30 $n \in \mathrm{N}$, then:

(i) $\{x \mid x = 3n\}$
(ii) $\{y \mid y = n^2\}$
(iii) there is no greatest prime, and then arrange them in ascending order.

4.2 Every function can be expressed as a polynomial; each constituent mononomial will be the product of a number of 1's and 0's, etc.

4.3 (i) Use (10) (10D).

4.5

(i)

A	B	C	A′.B.C	A + A′.B.C
1	1	1	0	1
0	1	1	1	1
1	0	1	0	1
1	1	0	0	1
1	0	0	0	1
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

(ii)

X	Y	Z	X + Y + Z	Y′ + Z′	(X + Y + Z) .(Y′ + Z′)
1	1	1	1	0	0
0	1	1	1	0	0
1	0	1	1	1	1
1	1	0	1	1	1
1	0	0	1	1	1
0	1	0	1	1	1
0	0	1	1	1	1
0	0	0	0	1	0

4.6 $X = P \cap Q' \cap R \cap S'$
$Y = P' \cap Q' \cap R' \cap S'$

4.8 (i) Show that $X + Y = 1$, and $X.Y = 0$
(ii) $1 = X.Y.Z + X'.Y.Z + X.Y'.Z + X.Y.Z' + X.Y'.Z' + X'.Y.Z' + X'.Y'.Z + X'.Y'.Z'$

and use $A.B.C + A.B.C' = A.B.(C + C') = A.B$, etc.

4.11 $A.B' + A'.B = 1$

$$(A.B' + A'.B)' = 1' = 0 \qquad \textbf{2.4}\ \text{(iv)}$$
$$(A' + B).(A + B') = 0$$
$$A.B + A'.B' = 0$$
$$A.(B')' + A'.(B') = 0$$
$$A = B' \qquad \textbf{2.39}\ \text{(ii)}$$

similarly for **4.10** (f).

4.15 (a) $A \cup B' \neq B \cup A'$
(b) If $A \rightarrow B$ then $A' \cup B = 1$ **4.14**
$A \cap B' = 0$ (12, 9, 10D)
and if $P \subseteq Q$,, $P \cap Q' = 0$

the algebraic conditions for material implication and for a subset are the same.

(c) Use **4.15** (b) and **3.9** (v), or

$A' \cup B = 1$	$B' \cup C = 1$	**4.14**
$B' \cap (A' \cup B) = B'$,	$B \cap (B' \cup C) = B$	(IVD)
$A' \cap B' = B'$,	$B \cap C = B$	(2, 4D, 6D)
$(A' \cap B') \cup (B \cap C) = B \cup B' = 1$		(IV, 4)
$(A' \cup B) \cap (A' \cup C) \cap (B' \cup B) \cap (B' \cup C) = 1$		**2.33**
$1.1.1.(A' \cup C) = 1$		**4.14** (4)
$A' \cup C = 1$		(3D)
$A \rightarrow C$		**4.14**

4.16 $(A \rightarrow B) = A' \cup B$ **4.14**

so $P \cup Q = (P' \rightarrow Q)$

and $R \cap S = (R' \cup S')'$

$= (R \rightarrow S')'$

The whole argument is false. $(A \rightarrow B)$ is a statement which has a meaning only when A and B are *statements*. $A \subseteq B$ is a statement which has meaning if A and B are *sets*. We can regard $\rightarrow$ as an operation, but not $\subseteq$. For in the application of our algebra to logic, the elements are statements, and $A \rightarrow B$ is a statement, and so the set of elements is closed (see **1.3**) under this operation. $\subseteq$ does not qualify, as $P \subseteq Q$ is also a statement, not a set.

4.18 (a)

$A \downarrow B = A.B' + A'.B + A'.B'$	**4.17**
$= A.B' + A'$	(2, 4, 3D)
$= A' + B'$	(1, 9, 11)
$= (A.B)'$	(12)

or use $(A \downarrow B)' = A.B$ from **4.17**.

(b)

$A \downarrow A = (A \cap A)'$	(a)
$= A'$	(7D)

(c)

$A \downarrow B = (A.B)'$	(a)
$= (B.A)'$	(1D)
$= B \downarrow A$	(a)

(d)

$(A \downarrow A) \downarrow (A \downarrow A) = (A') \downarrow (A')$	(b)
$= (A')'$	(b)
$= A$	(9)

(e) Use (b), (a), (12), (9).

(f) $\cup$ eliminated by using (e)

$\cap$,, ,, ,, $A \cap B = (A \downarrow B)'$ (a)

(h) $A' \downarrow (A' \downarrow B) = A' \downarrow B'$

$$(A \downarrow A) \downarrow \{(A' \downarrow B) \downarrow (A' \downarrow B)\} = (A \downarrow A) \downarrow (B \downarrow B)$$

$$(A \downarrow A) \downarrow [\{(A \downarrow A) \downarrow B\} \downarrow \{(A \downarrow A) \downarrow B\}] = (A \downarrow A) \downarrow (B \downarrow B)$$

4.19 (v)
$$\begin{aligned}(A \uparrow B) \uparrow C &= (A' \cap B') \uparrow C\\ &= (A' \cup B') \cap C'\\ &= (A \cup B) \cap C'\\ &= (A \cap C') \cup (B \cap C')\\ &= (A' \uparrow C) \cup (B' \uparrow C)\end{aligned}$$

(vi)
$$\begin{aligned}A \cup B &= (A' \cap B')'\\ &= (A \uparrow B)'\end{aligned}$$

(viii) $A \cap (A' \cup B) = A \cap B$
becomes $(A \uparrow A) \uparrow \{(A \uparrow A) \uparrow B\} = (A \uparrow A) \uparrow (B \uparrow B)$

4.20 If A is the statement 'Andrew tells the truth', etc.

$$\begin{aligned}A &= B.C & C &= E.F + E'.F'\\ D &= A + B & E &= A.B\\ B &= E.F' + E'.F & F &= (B.C)'\end{aligned}$$

$$B = D = F = 1$$
$$A = C = E = 0$$

4.21
$$\begin{aligned}1 &= (A \cup B) \cap (A' \cup C')\\ &= (A' \cap B')' \cap (A' \cup C')\\ &= (1)' \cap (A' \cup C')\\ &= 0 \cap (A' \cup C')\\ &= 0\end{aligned}$$

and the statements are inconsistent.

4.22 Let A be the statement 'Atlantia sends a contingent', etc., and we have

$$W.Y = 0 \quad \text{(i)}$$
$$A.D = 0 \quad \text{(ii)}$$
$$V.W.Z = 0 \quad \text{(iii)}$$
$$W'.C.D = 0 \quad \text{(iv)}$$
$$Z'.E = 0 \quad \text{(v)}$$
$$Z.B = 0 \quad \text{(vi)}$$
$$X.C = 0 \quad \text{(vii)}$$

from (v) and (vi) $\quad B.(Z'.E) + E.(B.Z) = 0$
$$B.E = 0$$
,, (ii) $\quad A.D = 0$

3 from among A, B, C, D, E = 1,

so $C = 1$

from (vii) $X = 0$

„ (i) and (iii) $V.Y.Z = 1$

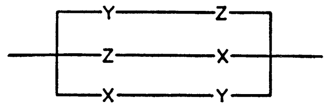
$V = Y = Z = 1$ (13D)

„ (vi) $B = 0$ so $E = 1$

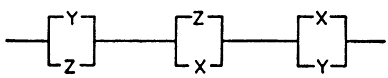
„ (iv) $D = 0$ so $A = 1$

5.5

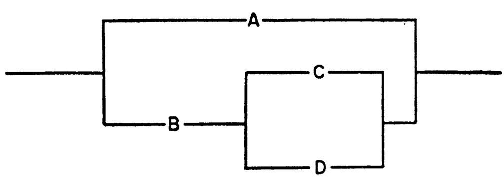
(i)



(ii)



(iii)



5.6 (i) $X.Y.Z + X' + X.Y.Z'$

$= X' + X.Y.(Z + Z')$

$= X' + X.Y$

$= X' + (X')'.Y$

$= X' + Y$ (11)

(ii) $A.B.C + B'.C.D + A'.B.C + B.C' + A.C.D'$

$= C.(B' + B'.D + A.B + A'.B + A.D')$

$= C.\{B' + B.(A + A') + B'.D + A.D'\}$

$= C.(B' + B + B'.D + A.D')$

$= C.(1 + B'.D + A.D')$

$= C(1)$

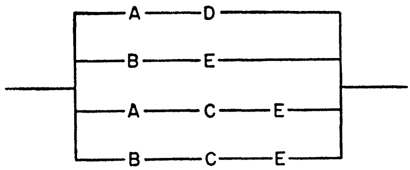
$= C$

5.7 (i) $A.B.C.D + A'.B.C.D + A.B.C'.D + A'.B.C'.D$
$= (A + A').B.C.D + (A + A').B.C'.D$
$= B.C.D + B.C'.D$
$= B.D.(C + C')$
$= B.D$

——— B ——— D ———

(ii) $X.Y.Z + X.Y.Z' + X.Y'.Z + X.Y'.Z' + X'.Y.Z + X'.Y.Z' + X'.Y'.Z$
$= X.Y.(Z + Z') + X.Y'.(Z + Z') + X'.Y.(Z + Z') + X.Y'.Z$
$= X.Y + X.Y' + X'.Y + X'.Y'.Z$
$= X.(Y + Y') + X'.(Y + Y'.Z)$
$= X + X'.(Y + Z)$
$= X + (Y + Z)$
$= X + Y + Z$

5.8

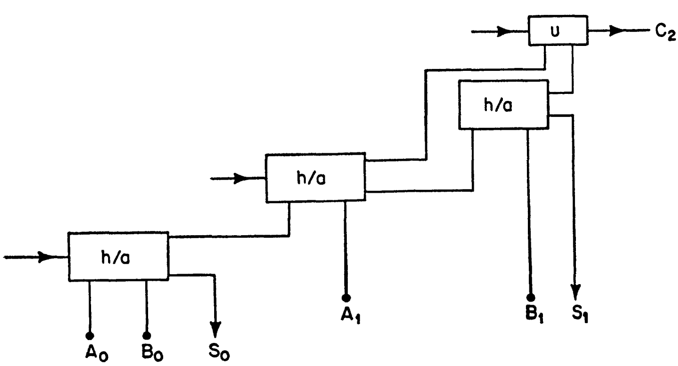


5.10 (i) 219 (ii) 11000011

5.16 ‘There is “one to carry” if both the digits are 1 (if $A_0.B_0 = 1$, then $A_0 = B_0 = 1$), but if, of A_0 and B_0, one and only one is equal to 1, then their sum is 1.’

5.18 Because the addition of numbers is commutative.

5.19



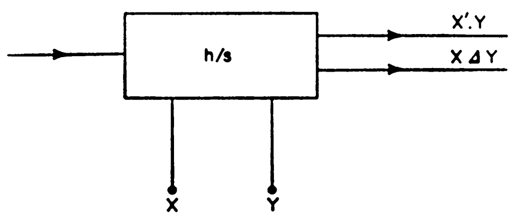
5.20

A_0	B_0	L_1	D_0
1	1	0	0
0	1	1	1
1	0	0	1
0	0	0	0

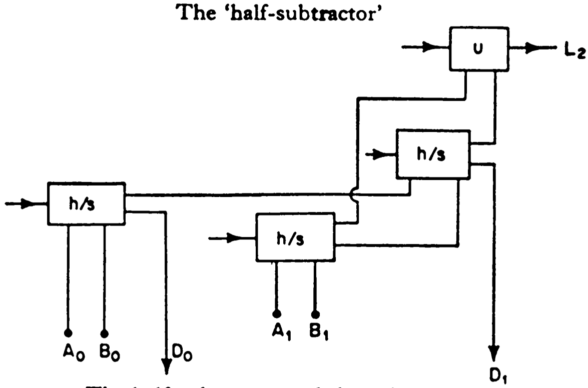
$$L_1 = A_0'.B_0, \qquad D_0 = A_0 \Delta B_0$$

A_1	B_1	L_1	L_2	D_1
1	1	1	1	1
0	1	1	1	0
1	0	1	0	0
1	1	0	0	0
1	0	0	0	1
0	1	0	1	1
0	0	1	1	1
0	0	0	0	0

$$L_2 = A_1'.B_1 + L_1.(A_1 \Delta B_1)$$
$$D_1 = L_1.(A_1 \Delta B_1)' + L_1'.(A_1 \Delta B_1)$$



The 'half-subtractor'



The half-subtractor and the subtractor

6.2 (i) 2/7 (ii) 1/2 (iii) 1/7

6.3 80 (there are two genders in French and three in German).

6.4 150/199, 24/199.

6.5 2^{N-1}.

6.6 The expression

$$(H + T).(H + T).(H + T).(H + T).(H + T)$$

can be regarded as a diagram of five coins, of which each must be a head or a tail. We can also regard H and T as numbers, and the array as representing $(H + T)^5$.

To every member of the soa ways of getting four heads and a tail, there is one and only one term in $H^4.T$ in the expansion of $(H + T)^5$, and conversely. But

$$(H + T)^5 = H^5 + 5.H^4.T + 10.H^3.T^2 + 10.H^2.T^3 + 5.H.T^4 + T^5$$

and so the probability of having at least four coins alike is

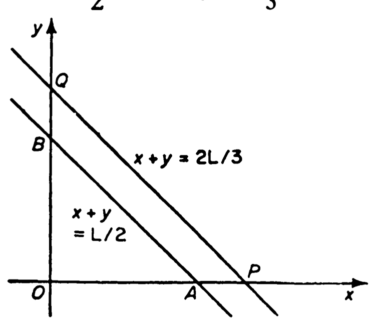
$$\frac{1 + 5 + 5 + 1}{1 + 5 + 10 + 10 + 5 + 1} = \frac{3}{8}$$

6.9 A bus in Mary's direction goes from John's stop 9 minutes after a bus in Dora's direction, then 3 minutes later is the next bus in Dora's direction; probability of visiting Mary is then

$$\frac{9 \text{ minutes}}{12 \text{ minutes}} = \frac{3}{4}$$

6.11 If x, y are the two shorter pieces of the stick, total length L, then

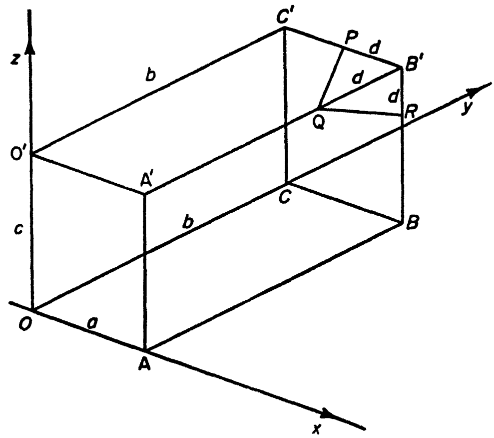
$$\frac{L}{2} < x + y < \frac{2L}{3}$$



so, by the method of **6.10**, the required probability is

$$\frac{\text{area of APQB}}{\text{area of OPQ}} = \frac{7}{16}$$

6.12 Axes as in diagram; there is a one-to-one relation between the times waited at stations and the points of the figure OABCC′B′A′D′.



For early arrival the point must lie below the plane PQR, i.e.

$$x + y + z = \mathrm{M}$$
$$a + b + c - \mathrm{M} = d$$

and probability is

$$\frac{\text{volume of OABRQPC'O'}}{\text{volume of OABCC'B'A'O'}} = \frac{6\,abc - d^3}{6\,abc}$$

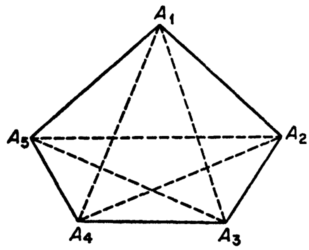
1.3/1 (i) yes (ii) yes (iii) no (iv) no

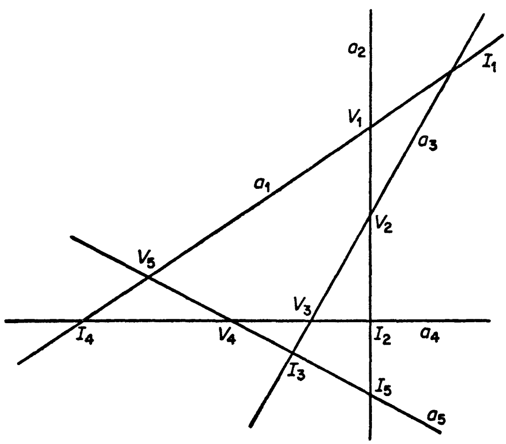
1.6/1 $\frac{x}{a}.\cos\frac{\alpha+\beta}{2} + \frac{y}{b}.\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2}$

$\cos(-\mathrm{A}) = \cos \mathrm{A}$

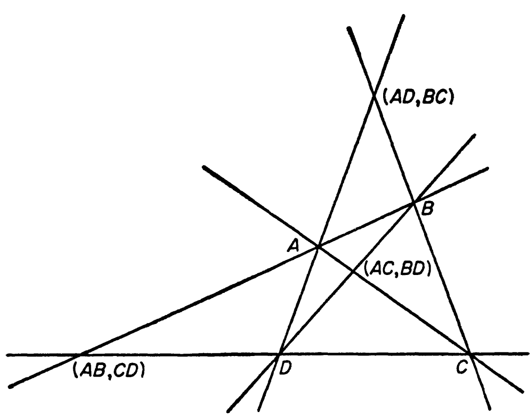
1.16/1 $(a + b).(a + c) = a + b.c$
$a^2 + a.(b + c) = a$ and $a \neq 0$
$a + b + c = 1$

1.22/1


A_1
A_2
A_3
A_4
A_5


a_2
I_1
V_1
a_3
a_1
V_2
V_5
V_3
I_4
V_4
I_2
a_4
I_3
I_5
a_5

1.22/2


(AD, BC)
B
A
(AC, BD)
(AB, CD)
D
C

1.25/1

$0'$	$= 0' + 0$	**1.23**
	$= 0 + 0'$	**1.4**
	$= 1$	**1.24**
$1'$	$= 1'.1$	**1.23**
	$= 1.1'$	**1.4**
	$= 0$	**1.24**

1.25/2 (i) $A \cap A = A$
(ii) $A \cap 0 = 0$
(iii) $(A \cap B) \cup (A \cap B') = A$
(iv) $(A \cup B \cup C') \cap (A \cup B \cup C) = A \cup B$
(v) $A \cap B \cap (A' \cup B') = 0$
(vi) $A \cup (A' \cap B) = A \cup B$

1.27/1

(I)	$a \mathrm{F} a$					true
(II)	If $a \mathrm{F} b$,	then	$b \mathrm{F} a$			false
(III)	If $a \mathrm{F} b$,	and	$b \mathrm{F} c$,	then	$a \mathrm{F} c$	true
(IV)	If $a \mathrm{F} b$,	then	$(a + c) \mathrm{F} (b + c)$			false
(IVD)	If $a \mathrm{F} b$,	then	$a.c \mathrm{F} b.c$			true

2.9/1 (i), (ii), (iii) use $P \cup (P \cap Q) = P$

2.16/1 (i) $C'.(A' + B')$
(ii) $X + Y + Z$
(iii) $A \cup B \cup C \cup D$
(iv) $X' \cap Y' \cap Z'$
(v) no simpler form
(vi) R

2.19/1 (i)

$$\begin{aligned} A.X &= A.Y \\ A'.X &= A'.Y \\ A.X + A'.X &= A.Y + A'.Y && \text{(IV)} \\ 1.X &= 1.Y \\ X &= Y \end{aligned}$$

(ii)

$$\begin{aligned} A + X &= A + Y \\ A' + X &= A' + Y \\ (A + X).(A' + X) &= (A + Y).(A' + Y) && \text{(IVD)} \\ X + A.A' &= Y + A.A' && \text{(2D)} \\ X + 0 &= Y + 0 \\ X &= Y \end{aligned}$$

2.29/1 (i) $B'.C' + C.A' + A'.B'$
(ii) $X' \cup Y$

2.30/2 (i) $X.Y + X.Y' + X'.Y$
(ii) same answer as **2.30**
(iii) $A.B.C + A.B.C' + A.B'.C' + A'.B'.C'$

2.30/4 (i) $A' + B$
(ii) 1
(iii) $P.R + Q'.(P'.R + P.R')$
(iv) $A'.B + C'.(A.B + A'.B')$

2.43/1 (i) $(A \Delta B) \Delta C$
$= (A.B' + A'.B).C' + (A.B + A'.B').C$
$= A.(B.C + B'.C') + A'.(B'.C + B.C')$
$=$ etc.
(ii) similar to (i)
(iii) see **2.39** (ii)

2.43/2 $Z' = A \Delta B \Delta C \Delta D$

3.2/1 (i) the soa strings of a violin
(ii) „ „ toes on a human foot
(iii) „ „ members of a hockey team
(iv) „ „ months of the year
(v) „ „ fish in the sea
(vi) „ „ prime numbers

3.2/2 $\dfrac{9!}{4!\,5!}$

3.4/1 (i) The soa members of the House of Lords
(ii) „ „ real numbers
(iii) {Tom, Dick, Harry, Jack, Jill}
(iv) $\{e\}$
(v) $\{2\}$

3.5/2 (a) I; (b) II; (c) II; (d), (e) III.

3.13/1 M = the soa racing motorists
T = „ „ profound thinkers
P = „ „ philosophers
Q = „ „ quick-witted men

$M \subseteq Q$, $\{\text{Plato}\} \subseteq T$, $P \subseteq T$, $Q.T = 0$, or $Q \subseteq T'$,
$\{\text{Plato}\} \subseteq T \subseteq Q' \subseteq M'$, and $P \subseteq T \subseteq \ldots$

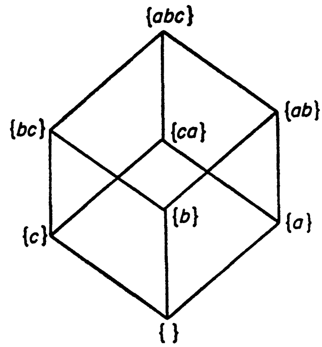
Plato was not a racing motorist, but it cannot be deduced that he was a philosopher.

3.13/3 (i) $\{x \mid x = 2^{n-1}\}$
(ii) $\{x \mid x = 3n - 2\}$
(iii) $\{x \mid x = n!\}$

3.13/5 (i) the soa triangular numbers
(ii) ,, ,, numbers numerically less than 1
(iii) ,, ,, positive numbers

3.13/6 (i) yes (ii) no

3.13/8 (i)



There are 8; from $\{pqrs\}$ to $\{p\}$, $\{q\}$, $\{r\}$, $\{s\}$, and from $\{pqr\}$ $\{pqs\}$ $\{prs\}$, $\{qrs\}$ to $\{\ \}$.
(ii) 2^n

3.13/9 In the diagram of **3.13/7**, for $\{p, q, r, s\}$ write (r, g, b, w) etc., and the point (r, b) represents red and blue full on; a line down the page, say from (r, g, b) to (r, b) thus represents a gradual subtraction of green from a combination of red, green, and blue; similarly (g) to (g, w) represents a gradual addition of white to green. A way of covering the whole figure without tracing any line twice is

$0, r, rg, g, 0, b, bg, g, gw, w, wr, r, rb, b, bw, rbw, rw, rgw, rg, rgb, rb, rbw, rbwg, rbg, bg, bgw, gw, rgw, rbgw, gbw, bw, w, 0$

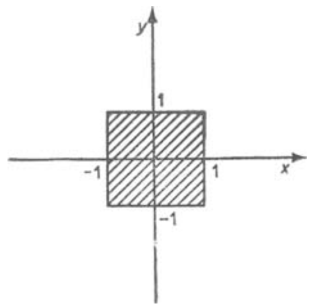
3.13/10
$$
\begin{aligned}
n(X \cap Y) &= 0, \quad z > 6r \\
&= 1, \quad z = 6r \\
&= 2, \quad 4r < z < 6r \\
&= 4, \quad z = 4r \\
&= 6, \quad 3r < z < 4r \\
&= 7, \quad z = 3r \\
&= 8, \quad z < 3r
\end{aligned}
$$

3.13/12 Symmetry about $y = x$

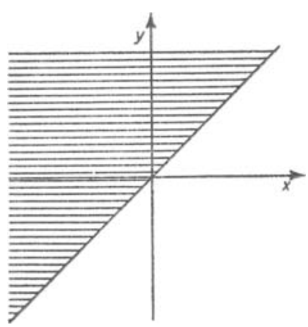
(i)	,,	,,	origin
(ii)	,,	,,	$x + y = 0$
(iii)	,,	,,	y-axis

3.13/13

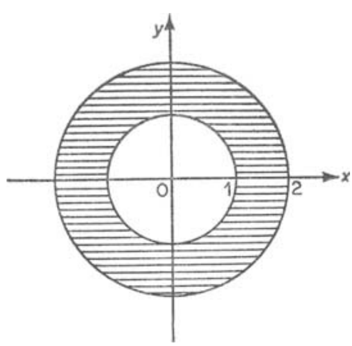
(i)



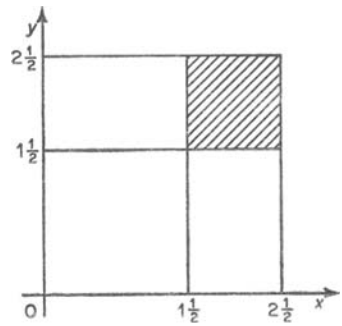
(ii)



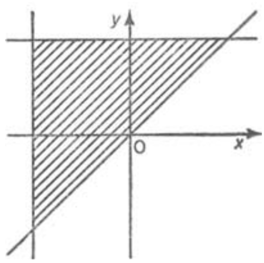
(iii)



(v)

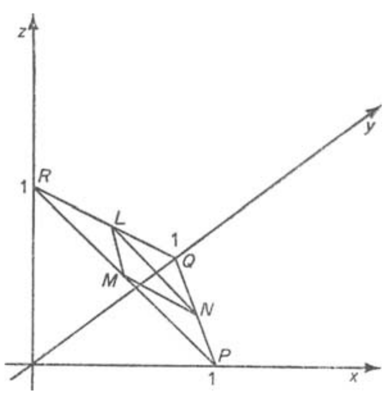


(vi)



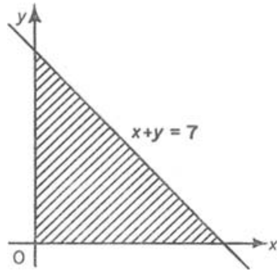
3.13/14 The point 4, 3

3.13/15

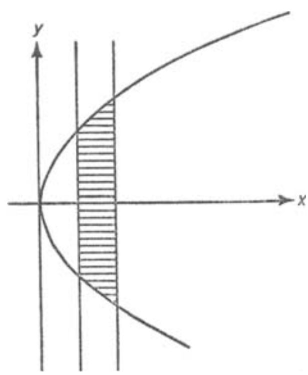


(A ∩ B) is the soa points in triangle PQR.
(A ∩ B ∩ C) is the soa points in the triangle LMN.

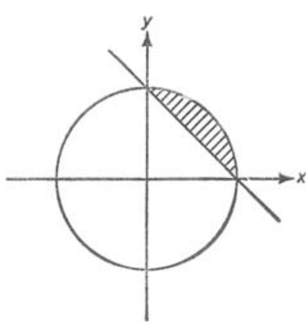
3.13/16 (i) $\{\langle x, y\rangle \mid x > 0, y > 0, x + y < 7\}$



(ii) $\{\langle x, y\rangle \mid a < x < 2a, y^2 < 4ax\}$



(iii) $\{\langle x, y\rangle \mid x^2 + y^2 < 1, x + y > 1\}$



3.13/17 The empty set.

3.17/1 E, F, G, T = the soa Europeans, fair, good-tempered, tall men, respectively.

$$\begin{array}{rlrl} E.F' + E'.F \subseteq G & & G'.(E.F' + E'.F) = 0 \\ E \subseteq (T + F) & & E.F'.T' = 0 \\ E.F' \subseteq T' & & E.F'.T = 0 \end{array}$$

so

$$E.F'.(T + T') = E.F' = 0$$
$$G'.E'.F = 0$$
$$G'.F \subseteq E$$

3.17/2 (i) All four tailors were members of each of the four sets mentioned.

(ii) An example of $A + A.B = A$.

(iii) The first two lines imply a universal soa little girls.

(iv) 99 out of a set of 100 selected dentists, or 99% of the soa dentists?

(v) C = the soa men called A. B. Charles
D = ,, ,, ,, at 73, Dover Road
E = ,, ,, ,, at Eastborough
F = ,, ,, ,, living in Kent

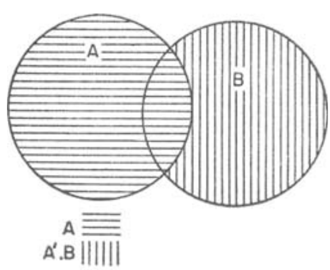
the addressee is a member of $C \cap D \cap E \cap F$

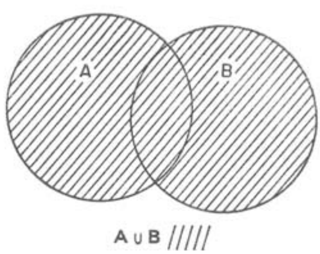
3.21/1 10

3.21/2 Use Venn diagram; the number of single, unemployed immigrants is -5.

3.32/3

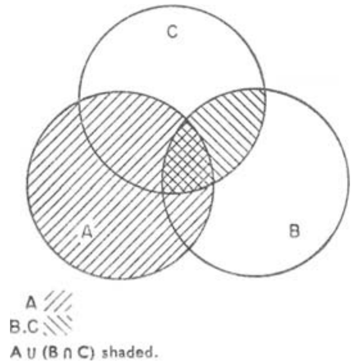
(i)

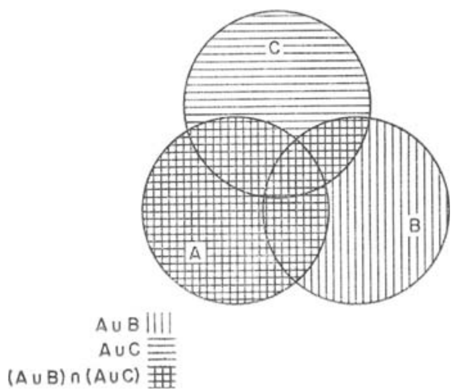




3.32/3

(ii)





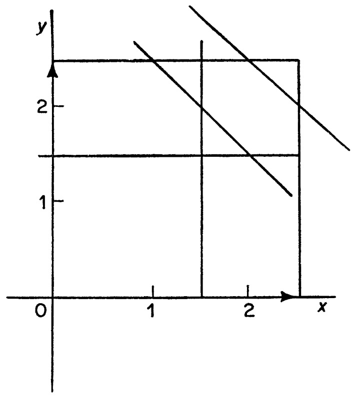
4.7/2 $X = A \Delta (B'.C')$
$Y = A.B' + A'.C'$
$Z = (A.B') \Delta C$

4.20/1 $E = 1, \quad A = B = C = D = F = 0$

5.27/1 $2^{13} - 1 = 8{,}191$

6.12/1 $\{\langle x, y\rangle \mid 3/2 < x, y < 5/2, 7/2 < x + y < 9/2\}$
$\{\langle x, y\rangle \mid 3/2 < x, y < 5/2\}$

probability $= 3/4$



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