Appendix

List of symbols

Symbols are defined locally in the text but this list is provided for convenience, and is arranged roughly in order of occurrence in the text.

E	=	Young's modulus
G	=	shear modulus
ν	=	Poisson's ratio
w	=	load per unit length
F	=	force (usually in a component)
P	=	load (usually imposed on a structure)
p	=	pressure (force per unit area)
ρ	=	density
σ	=	stress, usually tensile
R	=	radius, and sometimes reaction
D	=	diameter, and occasionally depth of a beam
d	=	diameter, and occasionally displacement
L	=	length
t	=	thickness (wall thickness where appropriate)
I	=	second moment of area
M	=	mass, and sometimes bending moment
U.T.S.	=	ultimate tensile strength (= failing load/original C.S.A.)
C.S.A.	=	cross-sectional area
S.C.F.	=	stress-concentration factor
I.C.	=	Instantaneous centre
θ, ϕ	=	angles
c.g.	=	centre of gravity (mass centre)
α	=	coefficient of linear thermal expansion, (expressed as p.p.m./°C)
k	=	thermal conductivity in SI units, W/m °C, i.e. heat flow in watts per square metre C.S.A. under a temperature gradient of 1 °C per metre.
M.P.	=	melting point
r	=	electrical resistivity, CGS version, $\mu\Omega$ cm, the resistance in microhms per cm length of a conductor of 1 cm^2 C.S.A. This old-fashioned CGS unit is used because electrical conductors of over 1 m^2 in cross-section are rare

c	=	speed of sound in solid rods (bar velocity)
r.p.m.	=	revolutions per minute
Hz	=	hertz (cycles per second)
a.c.	=	alternating current
d.c.	=	direct current
K	=	stiffness factor in torsion
δ	=	deflection
F.C.	=	flexural centre

In chapter 3 expressions are dimensionless and can be used with any consistent units. In chapter 6 SI units are used: MN/m^2 for E, U.T.S. and stresses generally except where stated otherwise. The British Standard preferred unit of N/mm^2 gives the same numerical values as MN/m^2.

A few conversion factors

1 standard atmosphere = 1013.25 mbar = 1.01325 bar = 101325 N/m^2
= 0.101325 MN/m^2 = 14.6959 lbf/in.^{2}
1 tonf/in.^{2} = 2240 lbf/in.^{2} = 15.443 MN/m^2
1 kilopound = 1000 lb mass or 1000 lbf (in United States)
1 kilopond (kp) = 1 kgf = 9.81 N (in Europe)
1 tonne (metric) = 0.984207 ton avoirdupois (long ton)
= 1.10231 U.S. (short) ton

Twist–bend buckling

The following is a simplified presentation of the twist–bend buckling situation. Figure 117a shows an I-section cantilever. It should be noted that ideal cantilevers tend to come in symmetrical pairs; a practical cantilever is longer than it seems because the end-fixing has some elasticity.

A down-load P carried by the cantilever is attached at a distance h from the centre We imagine giving the free end a deliberate small twist θ, which is shown greatly exaggerated.

Two effects appear, a twisting moment $Ph\theta$ and a sideways load component relative to axis A–A. This latter component causes a sideways deflection δ, where

$$\delta \approx \frac{P\theta L^3}{6EI_{sid}}$$

where E is Young's modulus for the material of the cantilever, I_{sid} is the second moment of the section in sideways bending, about axis A–A. The factor is 6 rather than 3 since the twist varies from θ down to zero at the fixing.

This gives a further twisting moment of $P\delta$ at the end, an average along the length of $\frac{5}{8}P\delta$. The total twisting moment T is given by

$$T = \tfrac{5}{8}P\delta + Ph\theta \qquad (1)$$

Such a moment will produce a twist angle of $TL/(KG)$, where K is the torsional stiffness constant of the section (see below) and G is the modulus of rigidity for the material.

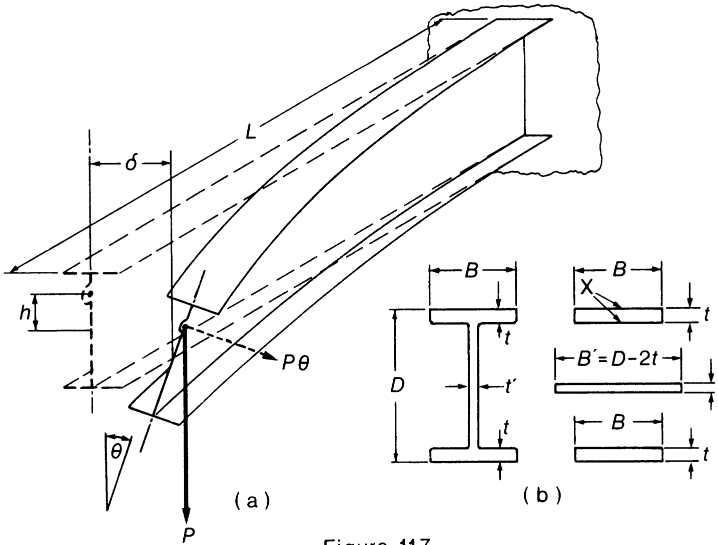

Figure 117

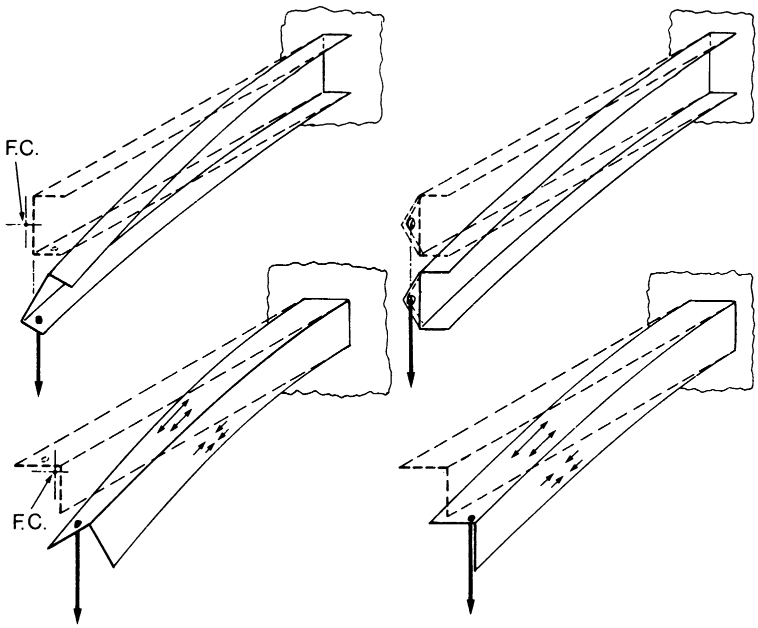

Figure 118

If we make the load P so great that the angle of twist produced by it is equal to the imposed twist θ, the twist becomes self-sustaining and mathematically indeterminate, indicating a buckling condition. In practice the twist may limit itself by the increase of K which comes with large deflections producing substantial changes of length; on the other hand it may become catastrophic through localised kinking.

$$\theta = \left(\frac{\frac{5}{8}P \times P\theta L^3}{6EI_{\text{sid}}} + Ph\theta\right)\frac{L}{KG} \tag{2}$$

Hence

$$KG = \frac{5P^2L^4}{48EI_{\text{sid}}} + PhL$$
$$= \frac{5P^2L^4}{48EI_{\text{sid}}}\left(1 + \frac{48hEI_{\text{sid}}}{5PL^3}\right) \tag{4}$$

Ignore the second term temporarily and find a temporary value of P, called P', where

$$P' = \frac{3.1}{L^2}\sqrt{(KGEI_{\text{sid}})}$$

This is substituted in the second term of equation 4 to give

$$KG = \frac{0.1P^2L^4}{EI_{\text{sid}}}\left[1 + \frac{3.1hEI_{\text{sid}}}{L\sqrt{(KGEI_{\text{sid}})}}\right] \tag{5}$$

From this we extract the value of P. If this is very different from P', we recycle it. Otherwise we obtain

$$P \approx \frac{3.1}{L^2}\sqrt{\left[\frac{KGEI_{\text{sid}}}{1 + (3.1h/L)\sqrt{(EI_{\text{sid}}/KG)}}\right]}$$

A much fuller solution gives a similar form with slightly higher numbers[64], so the present treatment errs on the safe side.

For a beam of length L with a central load and its ends restrained against twist but not against side-bending the load may be as above but with a numerical factor of perhaps 16. It must be emphasised that these are not design loads but loads at which collapse is extremely likely. Standard I-beams could begin to fail in this way before orthodox failure once the span exceeds about $20D$, $30B$ or $100t$.

The torsional rigidity of I-beams, channels, etc., is substantially that of all the flat strips of which it is composed. For example, consider the I-beam shown in figure 117b. This consists of three bars and since for a rectangular section of width B and thickness t

$$K = \tfrac{1}{3}Bt^3\left(1 - 0.63\frac{t}{B}\right)$$

the total is easily calculated.

Incidentally, the torsional shear stress in a rectangular bar is highest at point X, amounting to $(3B + 1.8t)/(B^2t^2)$ times the torque applied to the bar which would be about one-third of the torque applied to the I-beam shown. At other points it is lower, in the inverse ratio of the distance from the centre. In the complete I-beam the bending stresses are also likely to be important.

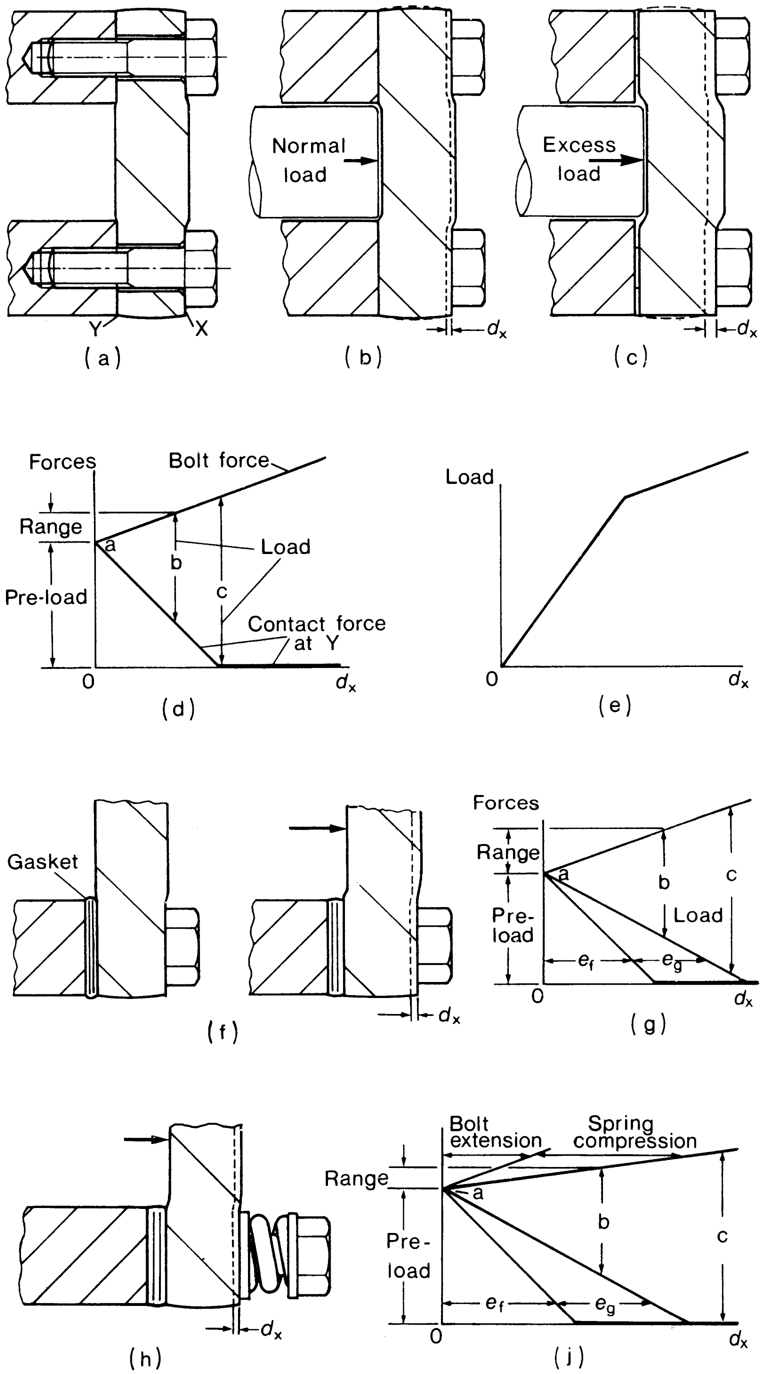
Y
X
(a)
Normal
load
(b)
Excess
load
(c)
d_x
Forces
Bolt force
Range
a
b
Load
c
Pre-load
Contact force
at Y
0
d_x
(d)
Load
0
d_x
(e)
Gasket
d_x
(f)
Forces
Range
a
b
c
Load
Pre-
load
e_f
e_g
0
d_x
(g)
d_x
(h)
Bolt
extension
Spring
compression
Range
a
b
c
Pre-
load
e_f
e_g
0
d_x
(j)

Figure 119

Unsymmetrical sections

Unsymmetrical sections have two properties which can be helpful or otherwise. They tend to twist unless the load-line passes through the flexural centre or shear centre. This phenomenon is connected with the flow of shear round the section. Angle sections have the further property that, loaded in the usual way, even if through the flexural centre, they deflect sideways as well as in the load direction. The tension side gets longer and is offset relative to the compression side which gets shorter. The result is some lateral curvature (figure 118).

Behaviour of bolted joints

Figure 119a shows a flanged joint, bolted and with initial tension. The effective stiffness of the bolts can be estimated; that of the flange and its surroundings is more difficult but will generally be substantially greater than the bolts holding it. Considering one bolt and its share of joint and loading: if a load is applied as shown be it mechanical or hydraulic, face X is displaced by an amount d_X, the bolt or stud elongates, and the flange relaxes and thickens, as shown (exaggerated) in figure 119b.

The load is resisted partly by the increased bolt force, partly by the reduced contact force at Y. This force cannot become negative, so eventually the faces part (figure 119c) and the bolt alone is effective. These relations are shown in figure 119d and the load–extension graph for the joint is plotted in figure 119e.

Figure 119f shows a joint with a gasket. Gaskets are usually required to prevent leakage of fluid and should be soft enough to settle into uneven gaps but also resilient enough to maintain a tight joint during changes of temperature, deflections due to load changes, creep etc. Figure 119g shows the effect of the same loads as in (b) and (c). The flange extension e_f and gasket extension e_g add up, making a larger total displacement of face X. The contact force at faces Y and Z has changed less than in (b) and (c), therefore the bolt-force must change more. For the same loads we now have a larger range of force and stress, which could matter if the load-cycle is repeated many times. The bigger displacement of X means that the structure is less rigid and may make more noise.

The bolt-force changes can be reduced by making the bolt more resilient, making it longer and slimmer, etc. (figure 79), or by use of a spring as in figure 119h. The assembly is even less rigid now so that face X deflects more than ever and separation occurs at a lower load than (c), but the bolt is treated more kindly since the displacement of X is taken up largely by the spring deflection. For the same normal bolt load, both the stress range and the stress at overloads are reduced (figure 119j).

If it is important to have a rigid assembly, the best arrangement would be a face-to-face joint with a resilient gasket set into a recess. This however is very demanding on the gasket material properties and is most likely to be successful in those cases where self-energising rubber seals can be used, supported by metal spring action.

Metal bellows expansion joints

Flexible metallic bellows are used as pipe expansion joints, accommodating length or angle changes. The flexibility resides mainly in the flat walls. Several thicknesses (plies) of metal can be used to resist higher pressures without loss of flexibility. The more corrugations, the more deflection can be allowed. For good fatigue life, makers state permissible extensions around 10 per cent and contractions up to, say, 15 per cent of the corrugated length. Angular movement is equivalent to extension at one side with contraction at the other; for convenience the permitted movement is expressed as an angle, perhaps $\pm \frac{1}{2}^{\circ}$ per corrugation. There is no obvious connection between the length and angle values but it should be noted that the proportions of corrugation pitch, inner diameter and radial width are relatively constant throughout any one range of designs.

Bellows are not only less stiff but also less strong axially then rigid pipes of the same diameter and pressure rating. In a pipe run as in figure 120a there is an obvious unbalanced force. If the pipes are not supported close to the joint, the bellows could overstretch and fail (figure 120b). In a straight run (figure 120c) there is no danger of this simple failure; however, the internal pressure acts on the flat walls of the corrugations, trying to extend the bellows and putting the bellows and the pipe between them into compression like a strut. This strut could fail as a fixed-fixed strut and pop out sideways, the energy coming from the supply pressure times the volume increase (Flixborough 1975). Stability of a single bellows has been analysed by Haringx,[65] that of a pipe run with two bellows by Newland.[66]

Sideways motion can be prevented without hindering the linear motion by strong guide-pillars or by an internal (or external) sleeve fixed at one end only. Axial limit stops can be included in many cases (see figure 120d). Bellows units can be purchased complete with restraining devices. This helps to prevent damage during transit or installation but the restraining devices supplied are not necessarily strong enough for severe service conditions such as pipe misalignment.

For large pipe movements the dog-leg layout can be employed (figure 120e). If there is enough room, pipe flexibility alone can be relied on to relieve the extension loads. Alternatively, bellows units can be used as hinges but must be protected from the unbalanced forces, preferably as shown in figure 120f – if the motion is sure to be in one plane only – or as in figure 120g, using a gimbal ring, if universal motion is needed. An internal sleeve may also be required if the corrugated length is large. Sometimes the swinging link is set up vertically where the pipeline crosses over a roadway.

Some interesting and/or useful theorems

Maxwell's reciprocal theorem

In a structure, considerations of energy can be used to show that if a load at a point A produces a particular deflection at a point B, then transferring the same load to B will produce the self-same deflection at A. This is shown in many textbooks on structures; what is not shown is why we should be interested in the deflection at A due to a load at B. The deflections we most need to know are the maxima at any point, for clearance reasons, and the deflections at a load point. The latter are useful for resonance estimates and for assessing the influence of resilient foundations, etc. The maximum deflections at a point usually occur when the load is at that point or not too far away; the main use of the theorem seems to be as an intermediate stage in calculating for rolling loads in redundant frames.

Speed for maximum power from a belt drive

In a belt drive the maximum tension is limited by the fatigue strength of the belt in tension and bending. The ratio between tight-side and slack-side tension is limited by frictional grip considerations, as explained in most textbooks on the theory of machines, etc. The power transmitted would be proportional to the speed alone in any given set-up if it were not for centrifugal force in the belt. This theorem shows that the highest power transmitted in a given set-up occurs at that speed which makes the centrifugal tension one-third of the total permissible tension.

Calling the tight-side tension T_1 and the slack-side tension T_2 we suppose that the set-up is just tight enough to prevent slipping so as to minimise total tension (an exaggerated assumption), giving $T_1 = nT_2$ where n is a ratio depending on the layout but not on the speed, U.

$$\text{Power transmitted} = (T_1 - T_2)\,U = T_1 U\left(1 - \frac{1}{n}\right)$$

The belt can only be allowed a certain maximum tension, T_{max}, which has to cover the driving tension T_1, a term for the bending which depends on pulley radii, T_{b}, and the centrifugal tension $T_{\text{c}} = wU^2/(g)$, w being belt mass per unit length. g is shown in brackets since it may not be needed, depending on the units system in which we are working. Thus

$$T_1 \leqslant T_{\text{max}} - T_{\text{b}} - T_{\text{c}}$$

$$\text{Power} \leqslant (T_{\text{max}} - T_{\text{b}} - wU^2/(g))\,U\left(1 - \frac{1}{n}\right)$$

To find the speed for maximum power, differentiate with respect to U and set to zero

$$T_{\text{max}} - T_{\text{b}} - 3wU^2/(g) = 0$$

$$T_{\text{c}} = \frac{(T_{\text{max}} - T_{\text{b}})}{3}$$

This speed could perhaps be realistic where very small pulleys for a given belt are used; in other situations the speed thus calculated is mugh higher than speeds usually recommended by belt manufacturers. Besides, the fatigue strength T_{max} and values for T_{b} are not readily available for proprietary belts – they must be derived backwards from catalogued power ratings. Finally it would be unwise to set up a belt drive so that the slack-side tension T_2 is only just sufficient.

Constantinesco's theorem

This theorem closely resembles the preceding one mathematically; it shows the highest power that a given pipe can deliver from a source at fixed pressure. The pressure loss in a pipe tends to vary as the square of the flow rate. Thus if there is a supply at pressure P, the pressure at the outlet will be $P - kQ^2$ where Q is the flow rate (please don't ask what to do with compressible fluids!).

$$\text{Power delivered} = (P - kQ^2)\,Q = PQ - kQ^3$$

Q is the only variable in this system, so maximum power requires that

$$\frac{\mathrm{d\,Power}}{\mathrm{d}Q} = 0 = P - 3kQ^2$$

Thus for maximum power we must use that flow rate that makes the friction loss one-third of the supply pressure.

It is not to be supposed that a one-third loss is a generally sensible value for designing pipes to transmit power at steady rates. Nevertheless the theorem is worth looking at since there are some situations where peak power is needed infrequently and the pipe size is quite important. One such case is in aircraft, where hydraulic actuation of control surfaces and undercarriages is used and the weight of long pipe-runs needs minimising. Another possible application may be in small hydro-power schemes for farms or villages. These do not necessarily need expensive dams; the turbines may be improvised from boat propellers or second-hand centrifugal pumps running backwards. The water pipe from some convenient stream may well be the major expense.

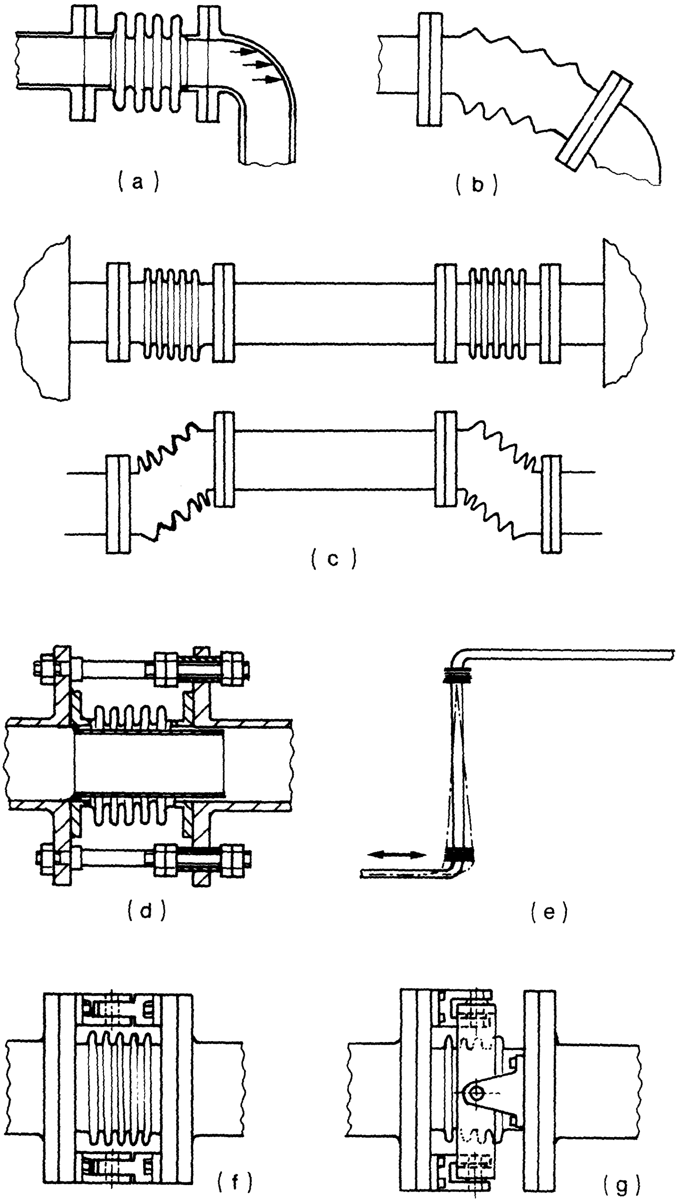

Figure 120

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