Lecture Notes in Statistics

Edited by J. Berger, S. Fienberg, J. Gani,
K. Krickeberg, I. Olkin, and B. Singer

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A Calculus for Factorial Arrangements

Springer-Verlag Berlin Heidelberg GmbH

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Mathematical Subject Classification: 62K15, 62K10

ISBN 978-0-387-97172-8 ISBN 978-1-4419-8730-3 (eBook)
DOI 10.1007/978-1-4419-8730-3

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Originally published by Springer-Verlag Berlin Heidelberg in 1989

2847/3140-543210 – Printed on acid-free paper

TO

Suman, Piyush, Gaorav
SG

My parents and wife
RM

ACKNOWLEDGEMENTS

Most parts of this monograph were written during the 1987-88 academic year when one of the authors (RM) was visiting the Department of Mathematical Sciences, Northern Illinois University. We are thankful to the department and our colleagues for a very friendly and congenial atmosphere during the preparation of this monograph. We also thank Professors W. T. Federer and J. N. Srivastava for some helpful comments.

The first author (SG) is grateful to his wife Suman and sons Piyush and Gaorav for their love, support and affection throughout. The second author (RM) wishes to record his deep gratitude to Professor S. K. Chatterjee, Department of Statistics, University of Calcutta, who had inspired and initiated him to this area of research. He is also grateful to his parents and wife for their love and support in various ways.

Finally we thank Mrs. Sara Clayton for her diligent typing of the monograph and Ms. Susan Gordon of Springer-Verlag Inc. for her help at various stages.

Sudhir Gupta
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