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A *Beginner's* Guide to Graph Theory

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for Denise and Carolyn

Preface

Because of its wide applicability, graph theory is one of the fast-growing areas of modern mathematics. Graphs arise as mathematical models in areas as diverse as management science, chemistry, resource planning, and computing. Moreover, the theory of graphs provides a spectrum of methods of proof and is a good training ground for pure mathematics. Thus, many colleges and universities provide a first course in graph theory that is intended primarily for mathematics majors but accessible to other students at the senior level. This text is intended for such a course.

I have presented this course many times. Over the years classes have included mainly mathematics and computer science majors, but there have been several engineers and occasional psychologists as well. Often undergraduate and graduate students are in the same class. Many instructors will no doubt find themselves with similar mixed groups.

It is to be expected that anyone enrolling in a senior level mathematics course will be comfortable with mathematical ideas and notation. In particular, I assume the reader is familiar with the basic concepts of set theory, has seen mathematical induction, and has a passing acquaintance with matrices and algebra. However, one cannot assume that the students in a first graph theory course will have a good knowledge of any specific advanced area. My reaction to this is to avoid too many specific prerequisites. The main requirement, namely a little mathematical maturity, may have been acquired in a variety of ways.

My students' reasons for studying graph theory have also been mixed. Some have seen graph theory as an area of pure mathematics to be studied for its own sake, others as an adjunct to such mathematical studies as combinatorics, algebra, or functional analysis, and others as an applied area. Even within a single area of application, there are diverse reasons: one electrical engineer, for example, may use graph theory to study circuits, while another may see it as a foundation for neural networks. Taking this into account, I have concentrated on the topics that appeal to the majority of users, and generally I have omitted those with a smaller readership. I hope that I have attained a balance between the theoretical and practical approaches. I have included several more specialized chapters dealing with material that students seem to enjoy, and (frankly) ones that I like to teach. Instructors can supplement the selection with other topics to meet their specific needs.

Outline of the topics

The first four chapters introduce the main ideas of graph theory and conclude with a short discussion of the minimal spanning tree problem. The idea is to introduce graph-theoretic reasoning along with an easy algorithm.

The fifth chapter deals with the application of vector space ideas to graphs. This is one of three specialized excursions, and could be omitted or deferred; in particular, anyone who has not seen a formal linear algebra course (including at least the general definition of a vector space) should probably skip this chapter. But I have found that students with an algebraic background often like this material, and if it is to be included at all, this is then probably the best place for it.

Chapter 6 explores another special topic, one-factorizations of graphs. All students should read the first section, and most will enjoy the second. The rest is a little specialized, but introduces some good examples of graph-theoretic reasoning.

There follows an exposition of coloring and planarity. A discussion of edge-coloring is included, and should particularly interest those who read all of Chapter 6. Ramsey's Theorem is studied in Chapter 9; the first section is of broad interest, while the general treatment given later will especially appeal to those with a wider combinatorial background. The later parts of this chapter are quite difficult.

Chapter 10 introduces directed graphs. The two following chapters are devoted to two important application areas that will appeal to students of management science, namely critical paths and network flows. Students who do not know a little statistical theory — enough to use the normal distribution, and to look up values in a table of the normal probability function — should skip Section 11.3.

A chapter on graph-theoretic algorithms concludes the book. I believe that computer scientists will see more than enough of these topics in other courses, and that graph algorithms are more appropriately studied among other algorithms, not among other aspects of graphs. Moreover, a proper study of algorithms would require some study of computational complexity, which would probably not interest the majority of readers. So my treatment here is intentionally short and quite superficial, but should satisfy the needs of those who are not likely to revisit the topic.

I thought of including several further topics of pure graph theory — covering theorems, line graphs, general problems on cycles, various extremal problems

— but rejected them because of their specialized appeal; three specialized topics (graphs and linear spaces, one-factorizations, Ramsey theory) should be enough, and these are my preferences anyway. A pure-mathematically minded instructor could easily replace Chapters 11 through 13 with other appropriate topics; of course, her/his interpretation of what is "appropriate" could certainly be different from mine. Excellent sources of such material are the texts by West [106][1] and Balakrishnan [4], introductions to the subject that go much deeper than we do here. An instructor who prefers a more applied course will find a rich fund of further material; some references are [19], [20] and [84].

Further reading

I have made frequent reference to the papers where results first appeared, and to the research literature in general. Those who want to go further into a topic can consult the papers cited. For general reading, the student may wish to consult one of the more advanced volumes on graph theory, such as [4] and [106]. Volumes of surveys of specific topics include three volumes edited by Beineke and Wilson [7, 8, 9] and two edited by Fulkerson [42, 43]. One very readable book is Tutte's *Connectivity in Graphs* [97], now 35 years old but still an excellent research resource. Yap's collection [109] of short monographs on three topics of graph theory includes an excellent introduction to edge-coloring. Haynes, Hedetneimi and Slater [60] have recently written a first-class introduction to a current hot topic, domination theory. The reader interested in graph matchings and factorization may wish to consult [71] or [104]. References to applications include [19], [20] and [84]. Biggs, Lloyd and Wilson provide a good deal of historical information, and some classical papers in [11].

The exercises

I have tried to include a reasonable number of problems, but not so many that the student becomes overwhelmed. They range from the easy to the difficult. In a few cases, hints are included, and there are answers and solutions to selected exercises. A hint is indicated by ^{H} preceding the exercise number, while ^{A} announces an answer or solution.

Acknowledgments

This book has grown out of graph theory courses that I have taught at the University of Newcastle and Southern Illinois University over the past 30 years. A number of students have made comments and contributions; I hope they will forgive me if I do not mention them by name, but if I tried to do so, I would surely (unintentionally) omit some.

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[1]Citations refer to the **References** section at the back of the book.

My friend and colleague Roger Eggleton used a draft version of the text for a course at Illinois State University. He made a large number of intelligent and informed comments and corrections, including the discovery of at least two instances where a widely-published, accepted "proof" was in need of amendment. I am very grateful to him and his students for their assistance.

Finally, the book would not exist without the support of George Anastassiou, of the University of Memphis, who recommended it to the publisher, and of Birkhäuser's Ann Kostant, who suggested the title, and Tom Grasso. Thank you all.

Contents

List of Figures