

A Brief Journey in Discrete Mathematics

Randolph Nelson

A Brief Journey
in Discrete Mathematics

 Springer

Randolph Nelson
(Home address)
Beverly, MA, USA

ISBN 978-3-030-37860-8 ISBN 978-3-030-37861-5 (eBook)
<https://doi.org/10.1007/978-3-030-37861-5>

Mathematics Subject Classification: 97N70

© Springer Nature Switzerland AG 2020

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors, and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, expressed or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Switzerland AG.

The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

*Beauty is the first test:
There is no permanent place in the world for
ugly mathematics.*

G. H. Hardy (1877–1947)

Dedicated to my wife - Cynthia Nelson

Acknowledgments

I would like to thank several of my colleagues at work who shared their enthusiasm and interest for the book over the many years it took to write. Sean McNeill suffered through many monologues concerning mathematical relationships that I found particularly beautiful and I remember Rod Dodson having to undergo similar orations. Eric Norman was always receptive to the ideas in the book and even had his daughter read some chapters. Linda Wu, a fellow mathematician, shared her exuberant interest in the material and continually encouraged me towards publication.

I have sat across a restaurant table literally hundreds of times with my business partner, Ira Leventhal. Frequently, during our conversations, the topic would veer away from business towards mathematics. Invariably, this led me to profit from Ira's uncanny intuition regarding numerical relationships.

I would be amiss without acknowledging two professors who have had a major influence on my research career. Rod Oldehoeft's warm and enthusiastic reception of a new student who walked into his office one day started my career. Leonard Kleinrock, my doctorate thesis advisor, opened my eyes to the joys of mathematical modeling with his unique and charismatic view of the subject which launched me on my own research path. I am forever grateful to these professors for their life changing guidance.

I would also like to thank the editor at Springer, Elizabeth Loew, who offered her encouragement for the book and guided its reviews. The last anonymous reviewer, who I wish to particularly thank, made several cogent criticisms that improved the book.

This book is dedicated to my wife, Cynthia, without whom, in so many ways, it would never have seen the light of day. Appreciation also goes to my children, Austin, Caresse, and Cristina, who sometimes lost their father even while he was physically present.

Contents

1	Introduction	1
2	Let Me Count the Ways	3
2.1	Permutations: With and Without Replacement	4
2.1.1	Derrangements	7
2.2	Combinations: Without Replacement	9
2.2.1	Binomial Identities	10
2.3	Combinations with Replacement	16
2.3.1	Binomial-R Identities	18
2.3.2	Polynomial Solutions to Combinatorial Problems	20
2.4	Transforms and Identities	23
3	Syntax Precedes Semantics	27
3.1	Stirling Numbers of the First Kind	29
3.2	Stirling Numbers of the Second Kind	32
3.2.1	The Stirling Transform and Inverse	35
3.3	Combinatorial Interpretation	36
4	Fearful Symmetry	39
4.1	Symmetric Functions	40
4.1.1	Simple Polynomials	40
4.1.2	The Quadratic Equation	41
4.1.3	Equation of the Minimum Distance Line	42
4.1.4	The Pythagorean Theorem	44
4.1.5	Cubic Polynomials	45
4.2	Elementary Symmetric Polynomials	48
4.2.1	Newton–Girard Formula	51
4.2.2	Identities and Combinatorial Coefficients....	52

- 4.2.3 Inclusion–Exclusion 55
- 4.3 Fundamental Theorem of Symmetric Polynomials ... 57
- 4.4 Galois’ Theorem and Numerical Solutions..... 59
- 5 All That Glitters Is Not Gold 63**
 - 5.1 The Golden Ratio 63
 - 5.1.1 Fibonacci Numbers 65
 - 5.1.2 A Closed Form Solution..... 67
 - 5.2 An Alternate Derivation 72
 - 5.3 Generalized Fibonacci Numbers 73
 - 5.4 k -Bonacci Numbers 74
 - 5.5 Generalization of the Fibonacci Recurrence 76
- 6 Heads I Win, Tails You Lose 79**
 - 6.1 The Mathematical Model 79
 - 6.1.1 Games That End Even..... 81
 - 6.1.2 Catalan Numbers 82
 - 6.1.3 Non-intuitive Results..... 84
 - 6.2 The Correct Insight..... 90
 - 6.3 Particular Sequences 91
 - 6.4 Conclusions..... 92
- 7 Sums of the Powers of Successive Integers 93**
 - 7.1 A General Equation 94
 - 7.1.1 Iterative Approach 100
 - 7.2 Triangular Numbers 102
 - 7.3 Cauchy’s Theorem 107
- 8 As Simple as $2 + 2 = 1$ 109**
 - 8.1 Modular Arithmetic 109
 - 8.2 Fermat’s Little Theorem 112
 - 8.3 Lagrange’s Theorem 114
 - 8.4 Wilson’s Theorem 115
 - 8.5 Cryptography 117
- 9 Hidden in Plain Sight 119**
 - 9.1 Properties of Prime Numbers..... 119
 - 9.1.1 Properties of Integer Divisors..... 122
 - 9.2 The Prime Counting Function 124
 - 9.3 There Is Always a Prime Between n and $2n$ 127
 - 9.3.1 The Prime Number Theorem with a Controversy 131

10 Running Off the Page..... 133

 10.1 Simple Continued Fractions..... 134

 10.1.1 Periodic Simple Continued Fractions 139

 10.1.2 Summary of Results 140

 10.2 General Method to Create a Continued Fraction 142

 10.2.1 Integer Quadratics and Quadratic Surds 143

 10.3 Approximations Using Continued Fractions 150

 10.3.1 Best Approximations 156

 10.4 Lagrange’s Theorem and Historical Review 158

A Tools of the Trade 161

 A.1 Recurrence Relationships 161

 A.2 Adding Zero to an Equation 163

 A.3 Induction 166

 A.4 Contradiction 169

 A.5 Order of Summations..... 171

B Notation and Identities Derived in the Book 173

Bibliography 181

Index 183