# **SpringerBriefs in Mathematical Physics**

### Volume 15

### Series editors

Nathanaël Berestycki, Cambridge, UK Mihalis Dafermos, Cambridge, UK Tohru Eguchi, Tokyo, Japan Atsuo Kuniba, Tokyo, Japan Matilde Marcolli, Pasadena, USA Bruno Nachtergaele, Davis, USA More information about this series at http://www.springer.com/series/11953

## Karen Yeats

# A Combinatorial Perspective on Quantum Field Theory



Karen Yeats Department of Mathematics Simon Fraser University Burnaby, BC Canada

and

Department of Combinatorics and Optimization University of Waterloo Waterloo, ON Canada

ISSN 2197-1757 ISSN 2197-1765 (electronic) SpringerBriefs in Mathematical Physics ISBN 978-3-319-47550-9 ISBN 978-3-319-47551-6 (eBook) DOI 10.1007/978-3-319-47551-6

Library of Congress Control Number: 2016953661

#### © The Author(s) 2017

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

# Acknowledgements

I would like to thank all of my colleagues, collaborators, and students, but particularly Dirk Kreimer from whom I learned the keys to this whole area and my students in summer 2016, Iain Crump, Benjamin Moore, Mohamed Laradji, Matthew Lynn, Wesley Chorney, and Maksym Neyra-Nesterenko, who helped proofread this brief. I would also like to thank Cameron Morland for his support.

## **Contents**

Part	t I P	reliminaries		
1	Intro	duction	3	
2		ntum Field Theory Set Up	5	
3	Combinatorial Classes and Rooted Trees			
_	3.1	Combinatorial Classes and Augmented	ç	
		Generating Functions	ç	
	3.2	Combinatorial Specifications and Combinatorial		
		Dyson-Schwinger Equations	14	
	Refe	rences.	18	
4	The Connes-Kreimer Hopf Algebra		19	
	4.1	Combinatorial Hopf Algebras	19	
	4.2	The Connes-Kreimer Hopf Algebra of Rooted Trees	25	
	4.3	Physical Properties	27	
	4.4	Abstract Properties	30	
	Refe	rences.	32	
5	Feynman Graphs			
	5.1	Half Edge Graphs	35	
	5.2	Combinatorial Physical Theories	37	
	5.3	Renormalization Hopf Algebras	40	
	5.4	Insertion and the Invariant Charge	42	
	5.5	Graph Theory Tools	48	
	5.6	Feynman Rules	50	
	Refe	rences	54	

viii Contents

Part	II :	Dyson-Schwinger Equations	
6		oduction to Dyson-Schwinger Equations	57 59
7	7.1 7.2	Hopf Algebras from Dyson-Schwinger Equations Simple Tree Classes Which Are Sub-Hopf More Physical Situations rences.	61 63 65
8		Factorial and Leading Log Toys	67 70
9	<b>Cho</b> r 9.1	rd Diagram Expansions	71
	9.2 9.3 9.4 Refer	Form  Rooted Connected Chord Diagrams  The $s = 2$ , $k = 1$ Result  Binary Trees and the General Result  rences	71 73 75 78 80
10	Diffe	rential Equations and the (Next-To) <sup>m</sup> Leading Log	
	10.1 10.2	The (Next-To) <sup>m</sup> Leading Log Expansions	81 81 82 84
Part	III	Feynman Periods	
11	-	man Integrals and Feynman Periods	87 91
12	12.1 12.2 12.3 12.4	Planar Duality: Fourier Transform Completion Schnetz Twist Products and Subdivergences rences.	93 93 94 94 95 96
13		nvariant with These Symmetries	97 99
14	14.1 14.2	pht	101 101 104 106

Contents ix

15	The $c_2$ Invariant	
16	Combinatorial Aspects of Some Integration Algorithms	
Ind	ex	117