

# Lecture Notes in Computer Science

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C.A. Floudas  
P.M. Pardalos

A Collection of Test Problems  
for Constrained Global  
Optimization Algorithms

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# Preface

A significant research activity has occurred in the area of global optimization in recent years. Many new theoretical, algorithmic, and computational contributions have resulted. Despite these numerous contributions, there still exists a lack of representative nonconvex test problems for constrained global optimization algorithms.

Test problems are of major importance for researchers interested in the algorithmic development. This book is motivated from the scarcity of global optimization test problems and represents the first systematic collection of test problems for evaluating and testing constrained global optimization algorithms. This collection includes problems arising in a variety of engineering applications, and test problems from published computational reports.

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C.A. Floudas and P.M. Pardalos

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