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Philipp Gubler

# A Bayesian Analysis of QCD Sum Rules

Doctoral Thesis accepted by  
Tokyo Institute of Technology, Tokyo, Japan

 Springer

*Author (Current Address)*  
Dr. Philipp Gubler  
Strangeness Nuclear Physics Laboratory  
RIKEN, Nishina Center  
Wako, Saitama  
Japan

*Supervisor*  
Prof. Makoto Oka  
Department of Physics  
Tokyo Institute of Technology  
Tokyo  
Japan

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*Denn unser Wissen ist Stückerk,  
und unser Weissagen ist Stückerk.*

(1. Korinther 13, 9)

# Supervisor's Foreword

Quantum chromodynamics (QCD) has been established as the fundamental theory of the strong interaction of elementary particles, as a part of the standard model. The elements of QCD are quarks and gluons, which have a special internal degree of freedom, called “color” and interact with one another under the principle of the quantum gauge theory. Although the theory looks very simple as written down in one line (Lagrangian), it generates mysteriously a large variety of phenomena through the interactions of hadrons, such as proton, neutron, pion, and so on, which are made of quarks and gluons. This field of research, called hadron physics, has been developed in the past several decades through the close collaboration of experimentalists and theorists.

This volume of Springer Theses Series is devoted to present a newly developed analysis method of hadron spectra from the first principle of QCD. One of the serious difficulties in describing structures and interactions of hadrons from QCD is that the interactions of quarks and gluons are too strong at low energies to be treated by perturbation theory. As a consequence, quarks and gluons are always “confined” into a colorless (color-neutral) entity at low density and temperature and thus cannot be isolated or directly examined in the laboratory. Not many non-perturbative methods of analyzing QCD are known, and the QCD sum rule is one such semi-analytic method, while lattice QCD is a popular numerical method that requires huge computer power.

Dr. Philipp Gubler, in collaboration with a few members of Tokyo Institute of Technology and Kyoto University, has developed a new method of computing the hadron spectrum using the QCD sum rule approach. The QCD sum rule method, invented by M. A. Shifman, A. I. Vainshtein, V. I. Zakharov in 1979, has been successful in extracting masses of hadrons. The conventional analysis method, however, requires an assumption about the form of the spectral function and thus cannot be applied to cases where the shape of the spectral function is not known. The new method is based on the Bayesian inference theory and called maximum entropy method (MEM), which provides us with the most probable spectral function from given information by QCD. Dr. Gubler has applied the method to analyses of various hadron spectra and confirmed that the method works well and

indeed is superior to the conventional method of extracting the hadron spectrum from the QCD sum rule.

One important application happens to be temperature dependence of the spectra of heavy quarkonia, i.e., bound states of a heavy quark and a heavy antiquark. The subject is related to a phase transition of QCD at high temperature. There, quarks and gluons are supposed to become de-confined and form a plasma-like matter. Such matter may have been created at the beginning of our Universe, just after the Big Bang, while on the Earth, high-energy collisions of heavy ions will produce such matter for a short period. In 1986, T. Matsui and H. Satz proposed that the spectrum of heavy quarkonia is drastically modified in the plasma-like matter so that the formation of such matter can be detected by observing the dilepton spectrum in heavy ion collisions. Thus, theoretical study of heavy quarkonia in QCD at finite temperature is very important. According to the present analysis, QCD sum rules show that the quarkonia peaks in the spectral functions dissolve at finite temperature consistently with the Matsui-Satz prediction, while the dissociation temperatures are found to depend on the individual states. This volume contains all the details of this analysis and also other applications of this new method. We expect to have further applications and developments of this method, some of which have been already published after the thesis is accepted.

Dr. Philipp Gubler completed the doctoral course at the Graduate School of Science and Engineering, Tokyo Institute of Technology, in March 2012. Tokyo Institute of Technology is the leading Japanese National University in the fields of science and engineering and marked its 130th anniversary in 2011. The Department of Physics is one of the largest department with about 70 faculty members, 200 undergraduate students, and 180 graduate students. We joined the Springer Theses project in 2011 and Dr. Gubler is the first winner of the honor of being selected from the 18 successful doctoral theses in the academic year from Department of Physics at Tokyo Institute of Technology. I am very happy to introduce his achievement for the doctoral degree and also feel very honored to have supervised his 5-year study at our graduate school.

Tokyo, October 2012

Makoto Oka



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It was possible for me to fully concentrate on my research during the 3 years of the doctor course owing to the financial support of the Japan Society for the Promotion of Science for Young Scientists, which also supported my visits to several international conferences abroad.

Last but not least, I want to thank my family and friends, both back in Switzerland and Japan, for their moral support, love, and friendship.

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